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Quasi-Static Analysis of Generalized SAW Transducers Using the Green's Function Method

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Abstract—Surface-wave interdigital transducers can be analyzed rigorously by using a Green's function method, though extensive numerical analysis is necessary to evaluate the various transducer properties. It is shown here that simple analytic relationships can be derived from the Green's function theory if bulk wave excitation and electrode interactions are assumed to be negligible. Solutions are obtained for the amplitude of launched surface waves, the transducer conductance and susceptance, and the reflection, transmission, and conversion coefficients. All of these quantities are related in a direct way to the electrostatic charge density on the transducer, and the theory takes full account of electrostatic effects. The theory is valid for almost arbitrary transducer geometries. A reciprocity relation is shown to be valid even when electrode interactions are included.

I. INTRODUCTION

THE behavior of surface acoustic wave (SAW) interdigital transducers is in general very complex, involving complicated electrostatic effects, the presence of a variety of acoustic modes, and "electrode interactions" which cause propagating waves to be perturbed by electrodes under which they pass. A further complication is the variety of features to be analyzed: the amplitude of waves generated, the transducer admittance, and for acoustic waves incident on the transducer, the scatter-

ing parameters for conversion, reflection, and transmission. The complexity of the problem has resulted in a variety of theoretical methods for analysis which mostly make simplifying assumptions. Almost always it is assumed that the only acoustic mode present is a nonleaky surface wave, and it is also common to ignore electrode interactions. These assumptions are made in the network theory of Smith *et al.* [1], [2], using an array of bulk wave transducers as an analogy to the SAW case. This approach gives full information, including the transducer admittance and the scattering parameters for reception. The simpler and more convenient delta-function model of Tancrell and Holland [3] gives more limited information, though Hartmann's impulse model [4] extends this type of approach to enable calculation of the transducer admittance.

However, these methods do not account accurately for electrostatic effects. These effects must be modeled accurately in order to analyze transducer harmonic responses, withdrawal-weighted transducers or transducers with arbitrary polarity sequences, or for analysis of end effects. The electrostatic solution for a periodic single-electrode transducer was found by Engan [5], who later found solutions for other periodic structures [6], while numerical techniques have been developed for analysis of end effects and withdrawal-weighted transducers [7], [8], [21]. The electrostatic solution itself only gives the wave amplitudes launched by a transducer. To obtain other transducer properties, Smith and Peddler [9] de-

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veloped a network approach based on the electrostatic solution, though considerable complexity was introduced. More rigorously, a normal mode theory was used by Auld and Kino [10], [11] to relate the transducer conductance to the electrostatic solution, and Wagers [12] extended this to give the admittance for more general transducer structures.

The Green's function method of Milsom *et al.* [13], [14] is more general than the above approaches, taking full account of electrostatic effects, electrode interactions, and the various acoustic modes which can be present and giving full information on transducer properties for both launching and reception. This method is not convenient to use, however, as it involves complex and time-consuming numerical procedures. In this paper we consider the implications of the Green's function approach for cases where bulk wave excitation can be assumed to be negligible and the surface wave is nonleaky, conditions which were assumed by all the above authors except Milsom *et al.* Using relationships derived in [13] it is shown that the amplitude of generated surface waves, the transducer admittance, and the scattering coefficients for reception of surface waves can all be related to the electrostatic solution in a simple direct manner. Thus by using approximations which are usually valid in practice, the Green's function theory yields simple analytic expressions, and the complexity of the full theory is avoided. It is assumed generally that electrode interactions are negligible, though it is shown that a reciprocity relation can be derived without making this assumption. Since the analysis is an approximate form of the comprehensive Green's function method, it rests on a firm theoretical basis and gives physical insight into the transducer behavior. Practical use of the results given requires that the electrostatic problem must also be solved. This problem is not considered here, though several numerical techniques have been developed to solve it for arbitrary transducer geometries [7]–[9], [13], [21]. Moreover, for most surface-wave transducers the electrostatic solution can be found quite straightforwardly by using the superposition principle [15]–[17], giving accurate results without requiring iterative numerical methods.

The electrostatic solution is expressed here in terms of the charge density on the electrodes, denoted $\sigma_e(x)$. It could alternatively be expressed as the tangential electric field or the potential at the surface, since these three quantities are related in a simple way [7]. There are, however, two advantages in choosing the charge density. First, the transducer capacity is related quite simply to the charge density. Second, the surface wave amplitude generated by a transducer will be shown to be given by the Fourier transform of the charge density. This relationship emerges in a direct way because the charge density is finite only on one side of the observation point, which is outside the transducer. The electrostatic field and potential are in general nonzero on both sides of the observation point, so that the derivation of the Fourier transform relation is less clear if these quantities are used.

Section II introduces the first-order quasi-static approximation and analyzes a transducer launching surface waves, giving the wave amplitudes and the transducer capacitance and conductance. Section III analyzes reception of surface waves by

a transducer, giving the conversion, reflection, and transmission coefficients, and also a scattering matrix. Section IV gives a higher order approximation and derives the transducer admittance, including the acoustic susceptance $B_a(\omega)$ which is not given by the method of Section II.

The transducer is taken to have two bus-bars and to have no geometrical variation in the transverse direction, parallel to the SAW wavefronts. The geometry is otherwise arbitrary. In the main part of the paper it is assumed where appropriate that each electrode is connected to one of the bus-bars. Appendix C shows how the theory can be generalized to include disconnected electrodes.

II. LAUNCHING TRANSDUCER

In this section the quasi-static theory is developed for a transducer launching surface waves. Some basic relationships are first introduced in Section II-A, and the quasi-static approximation itself is introduced in Section II-B, which is concerned with the surface wave amplitudes generated by the transducer.

A. Basic Equations

We consider a semi-infinite piezoelectric with a plane surface, with a set of infinitely thin electrodes deposited on the surface. The electrodes are taken to have negligible sheet resistivity. The x and y directions are taken to be in the surface, with SAW propagation in the $\pm x$ directions. Variations in the y -direction are taken to be insignificant, so that the electrode geometry is independent of y , there is no apodization, and diffraction can be ignored. Under these conditions, the charge density $\sigma(x, \omega)$ and potential $\phi(x, \omega)$ at the surface are related, at frequency ω , by a convolution equation [13] involving the Green's function $G(x)$:

$$\begin{aligned} \phi(x, \omega) &= \int_{-\infty}^{\infty} G(x - x') \cdot \sigma(x', \omega) dx' \\ &= G(x) * \sigma(x, \omega) \end{aligned} \quad (1)$$

where the asterisk indicates convolution. Here $\phi(x, \omega)$ and $\sigma(x, \omega)$ are assumed to vary as $\exp(j\omega t)$, and this factor has been dropped. The charge density $\sigma(x, \omega)$ includes charges on the vacuum side of the electrodes as well as on the piezoelectric side and is zero except where an electrode is present. The Green's function $G(x)$ is related to [13] the effective permittivity $\epsilon_e(s)$ which is a function of slowness $s = k/\omega$. These functions are derived from the piezoelectric tensor of the substrate material. The use of (1) takes full account of all the piezoelectrically coupled acoustic modes, including where appropriate surface waves, leaky surface waves, and bulk waves, which all enter via the Green's function $G(x)$. Electrostatic effects are also included. The electric field is assumed to be quasi-static, that is, it is unaffected by the magnetic field.

Equation (1) can be used to calculate the behavior of almost arbitrary transducers for both launching and receiving surface waves. While the method is very general, it is not convenient to use. There are several reasons for this. First, the Green's function $G(x)$ must be found numerically for each material of interest. Secondly, $G(x)$ is a function of frequency ω , so that

(1) must be solved for one frequency at a time, and hence $\sigma(x, \omega)$ and $\phi(x, \omega)$ are frequency-dependent. A third factor is the mixed boundary conditions. In a launching transducer, $\phi(x, \omega)$ is known only at the electrode locations, and the solution must be found subject to the additional condition that $\sigma(x, \omega)$ is zero in the gaps. In the quasi-static approximation used here, the Green's function is approximated by an analytic form, and $\sigma(x, \omega)$ is approximated by the electrostatic form which is independent of frequency, so that much of the complexity is removed.

It has been shown [13] that $G(x)$ can be written as a sum of three terms representing electrostatic, surface wave, and bulk wave effects, respectively. Here it is assumed that bulk wave effects are insignificant, so that the bulk wave term can be omitted from the Green's function, which becomes a sum of an electrostatic term $G_e(x)$ and a surface-wave term $G_{SAW}(x)$:

$$G(x) = G_e(x) + G_{SAW}(x) \quad (2)$$

where

$$G_e(x) = -\ln |x| / [\pi \epsilon_s(\infty)]$$

$$G_{SAW}(x) = -jG_s \cdot \exp(-jk_0|x|). \quad (3)$$

Here $\epsilon_s(\infty)$ is the effective permittivity at infinite slowness, and $k_0 = \omega/v_0$ is the SAW wavenumber; v_0 is the free-surface SAW velocity. G_s is a negative constant related to the slope of the permittivity $\epsilon_s(s)$ at the surface-wave slowness $s_0 = 1/v_0$. It is shown in Appendix A that, as a consequence of ignoring bulk wave excitation, G_s is given by

$$G_s \approx \frac{1}{\epsilon_s(\infty)} \cdot \frac{v_m - v_0}{v_0}$$

where $v_m < v_0$ is the SAW velocity for a metallized surface. Thus G_s is a measure of the piezoelectric coupling strength of the substrate material for surface waves. By comparing the results of the theory below with those of the normal mode theory [10], [11], it can be seen that the constant $\epsilon_s(\infty)$ is given approximately by

$$\epsilon_s(\infty) \approx \epsilon_0 + \epsilon_p^T$$

with

$$\epsilon_p = (\epsilon_{xx}\epsilon_{zz} - \epsilon_{xz}^2)^{1/2}$$

where z is the normal to the surface and the superscript T indicates that ϵ_p^T is to be evaluated at constant stress.

B. Surface Wave Amplitudes

Consider now an isolated transducer launching surface waves. It is assumed that no acoustic waves are incident on the transducer. A potential V_T is applied across the transducer and as a result a charge density $\sigma(x, \omega)$ is produced. The corresponding potential $\phi(x, \omega)$, given by (1) and (2), is a sum of an electrostatic term arising from $G_e(x)$ and a surface wave term arising from $G_{SAW}(x)$.

We consider the potential at a point outside the transducer. The electrostatic term is not normally of interest here, and usually it is small because the electrostatic field decays away from the transducer while the SAW term is sinusoidal with

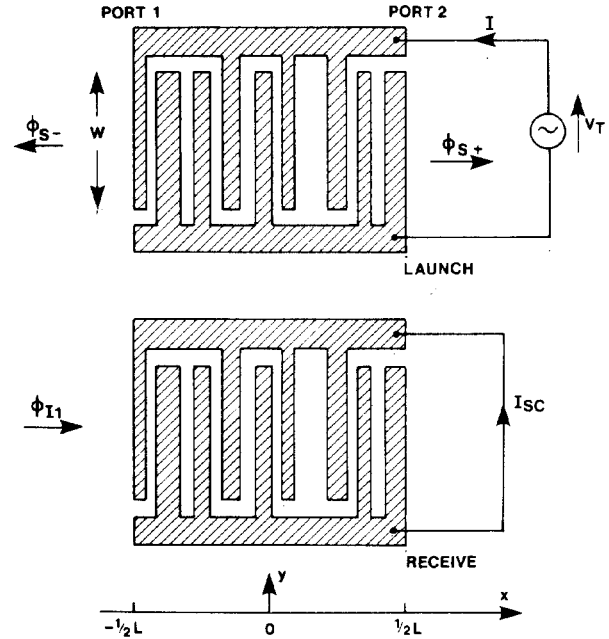


Fig. 1. Launching transducer and short-circuited receiving transducer.

x . The surface wave term is denoted $\phi_s(x, \omega)$ and is given by

$$\phi_s(x, \omega) = G_{SAW}(x) * \sigma(x, \omega). \quad (4)$$

The transducer is taken to occupy the region $|x| \leq \frac{1}{2}L$ as in Fig. 1. For $x < -\frac{1}{2}L$, the surface wave potential ϕ_{s-} refers to a wave traveling in the $-x$ direction, and from (3) and (4) is given by

$$\begin{aligned} \phi_{s-}(x, \omega) = & -jG_s \exp(jk_0x) \int_{-\infty}^{\infty} \sigma(x', \omega) \\ & \cdot \exp(-jk_0x') dx'. \end{aligned} \quad (5)$$

In this equation the modulus in (3) has disappeared because the charge $\sigma(x, \omega)$ is nonzero only for values of x to one side of the point where the potential is specified. Similarly, for $x > \frac{1}{2}L$ the surface wave potential, denoted $\phi_{s+}(x, \omega)$, gives the wave radiated in the $+x$ direction:

$$\phi_{s+}(x, \omega) = -jG_s \exp(-jk_0x) \int_{-\infty}^{\infty} \sigma(x', \omega) \exp(jk_0x') dx'. \quad (6)$$

In these equations the charge density $\sigma(x, \omega)$ must be found by solving (1) and (2), using the known potentials on the electrodes. This involves both the Green's functions $G_e(x)$ and $G_{SAW}(x)$. However, within the transducer the electrostatic Green's function $G_e(x)$ is dominant. In the *quasi-static approximation* the charge density is approximated by the solution obtained by ignoring the surface wave term. This solution is denoted $V_T \cdot \sigma_e(x)$, where $\sigma_e(x)$ is the electrostatic solution when unit potential is applied across the transducer. Thus, from (1) and (2), $\sigma_e(x)$ is defined as the solution of

$$\phi(x) = G_e(x) * \sigma_e(x) \cdot V_T \quad (7)$$

with the boundary conditions $\phi(x) = \pm \frac{1}{2} V_T$ on the electrodes and $\sigma_e(x) = 0$ in the gaps; a constant may be added to $\phi(x)$ without affecting $\sigma_e(x)$. Note that $\sigma_e(x)$ is real and independent of frequency.

Using $V_T \cdot \sigma_e(x)$ as the charge density in (5), the surface wave potential for $x < -\frac{1}{2}L$ is given, to first order, by

$$\phi_{s-}(x, \omega) \approx -jG_s \exp(jk_0 x) \int_{-\infty}^{\infty} V_T \sigma_e(x') \cdot \exp(-jk_0 x') dx' \quad (8)$$

This is essentially the Fourier transform of $\sigma_e(x)$, which is denoted $\bar{\sigma}_e(k)$ and given by

$$\bar{\sigma}_e(k) = \int_{-\infty}^{\infty} \sigma_e(x) \cdot \exp(-jkx) dx \quad (9)$$

with the inverse transform

$$\sigma_e(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{\sigma}_e(k) \cdot \exp(jkx) dk \quad (10)$$

Thus, in the quasi-static approximation,

$$\phi_{s-}(x, \omega) = -jG_s V_T \exp(jk_0 x) \cdot \bar{\sigma}_e(k_0), \quad \text{for } x < -\frac{1}{2}L \quad (11)$$

and from (6),

$$\phi_{s+}(x, \omega) = -jG_s V_T \exp(-jk_0 x) \cdot \bar{\sigma}_e^*(k_0), \quad \text{for } x > \frac{1}{2}L \quad (12)$$

where we have used the fact that $\bar{\sigma}_e(-k_0) = \bar{\sigma}_e^*(k_0)$, since $\sigma_e(x)$ is real.

It is convenient to define also the potential amplitudes of the surface waves measured at the two ends of the transducer, $x = \pm \frac{1}{2}L$. The ends are designated as ports 1 and 2, as on Fig. 1, with the corresponding SAW amplitudes denoted $\phi_{s1}(\omega)$ and $\phi_{s2}(\omega)$, respectively. From (11) and (12)

$$\phi_{s1}(\omega) = \phi_{s-}(-\frac{1}{2}L, \omega) = -jG_s V_T \bar{\sigma}_e(k_0) \cdot \exp(-\frac{1}{2}jk_0 L) \quad (13)$$

$$\phi_{s2}(\omega) = \phi_{s+}(\frac{1}{2}L, \omega) = -jG_s V_T \bar{\sigma}_e^*(k_0) \cdot \exp(-\frac{1}{2}jk_0 L) \quad (14)$$

Note that in the quasi-static case it is no longer necessary to solve for the charge density at each individual frequency, since $\sigma_e(x)$ is independent of frequency. Moreover, (8) shows that at each point in the transducer the charge density $V_T \cdot \sigma_e(x)$ can be regarded as a distributed source of surface waves, which propagate out of the transducer unaffected by the presence of any electrodes which they pass under. The quasi-static approximation is thus essentially equivalent to ignoring electrode interactions.

C. Transducer Admittance

It is assumed here that the transducer has two bus-bars, with each electrode connected to one of the bus-bars, as in Fig. 1. For an isolated transducer, with no acoustic waves incident

on it, the admittance measured between the two bus-bars is denoted $Y_T(\omega)$ and is written as a sum of three terms:

$$Y_T(\omega) = G_a(\omega) + jB_a(\omega) + j\omega C_T \quad (15)$$

where G_a , B_a , and C_T are real. Here G_a and B_a are terms due to the SAW excitation. C_T is defined as the capacity corresponding to the electrostatic charge density $\sigma_e(x)$ and can be written

$$C_T = \frac{1}{2} W \int_{-\infty}^{\infty} P(x) \cdot \sigma_e(x) dx \quad (16)$$

where W is the transducer aperture and $P(x)$ is a function giving the electrode polarities, defined by

$$P(x) = 1 \quad \text{if } x \text{ is on an electrode connected to the positive bus-bar.}$$

$$P(x) = -1 \quad \text{if } x \text{ is on an electrode connected to the negative bus-bar.} \quad (17)$$

At other values of x , in the interelectrode gaps and outside the transducer, the value of $P(x)$ is irrelevant because $\sigma_e(x)$ is zero.

The conductance $G_a(\omega)$ in (15) can be obtained from the power carried by the radiated surface waves. This follows because acoustic bulk waves, transducer ohmic losses, and electromagnetic radiation are all assumed to be absent, so that the radiated surface waves account for the power extracted by the transducer from the voltage source applied to it. A surface wave accompanied by a potential ϕ_s carries a power flow given by [13]

$$P_s = -\frac{1}{4} \omega W |\phi_s|^2 / G_s \quad (18)$$

Using (13) and (14), this gives the equal powers carried by the two radiated surface waves. Equating the sum of these powers to the power extracted from the source gives

$$G_a(\omega) = -\omega W G_s |\bar{\sigma}_e(k_0)|^2 \quad (19)$$

An alternative approach is to consider the current I taken by the transducer when a voltage V_T is applied. In the quasi-static approximation, the charge density on the electrodes is taken as $V_T \cdot \sigma_e(x)$. By summing the current flow into individual electrodes to give the total current I , the admittance is found to be $Y_T = I/V_T = j\omega C_T$, with C_T given by (16). Thus the quasi-static approximation is incapable of giving the conductance $G_a(\omega)$ and susceptance $B_a(\omega)$ by this approach. It is, however, shown in Section IV that a higher order approximation for the charge density can be used, and (19) is then obtained by calculating the current. An expression for $B_a(\omega)$ is also derived. In addition, the surface wave amplitudes given by (13) and (14) are still valid in the higher order approximation.

To evaluate the admittance of a given transducer, $\sigma_e(x)$ and $\bar{\sigma}_e(k)$ must first be found, and in general numerical techniques are necessary. However, in the particular case of the periodic single-electrode transducer (two electrodes per period), Engan [5] has given exact analytic expressions. At the center frequency of the fundamental response, $k_0 = \pi/p$, where p is the periodicity of the electrodes. At this frequency, the trans-

form of the charge density is, from Engan's equation (6b),

$$\bar{\sigma}_e(\pi/p) = \pi N \epsilon_s(\infty) / K(\kappa') \quad (20)$$

where the transducer is taken to be N periods long, and end effects are ignored. $K(\kappa')$ is an elliptic integral with argument $\kappa' = \cos(\frac{1}{2}\pi a/p)$, where a is the electrode width. The term $\epsilon_s(\infty)$ arises because our electrostatic formulation, (7) and (3), corresponds to Engan's, except that $\epsilon_s(\infty)$ replaces Engan's $(\epsilon_0 + \epsilon_{33}\epsilon_r)$. Using (19),

$$G_a(\omega_0) = -\omega_0 W G_s N^2 \pi^2 [\epsilon_s(\infty) / K(\kappa')]^2 \quad (21)$$

where $\omega_0 = \pi v_0/p$ is the center frequency of the fundamental response. In addition the capacity C_T is [5]

$$C_T = N W \epsilon_s(\infty) \cdot K(\kappa) / K(\kappa') \quad (22)$$

where $\kappa = (1 - \kappa'^2)^{1/2}$. Using the fact that $G_s = -|\Delta v/v|/\epsilon_s(\infty)$, as shown in Appendix A, we have

$$\frac{G_a(\omega_0)}{\omega_0 C_T} = \frac{N \pi^2 |\Delta v/v|}{K(\kappa) \cdot K(\kappa')} \quad (23)$$

This is identical to a result found using normal mode theory [10, (81)] and also agrees with formulae obtained from relationships for multistrip couplers [6]. For unity mark-to-space ratio ($a/p = \frac{1}{2}$), (23) gives

$$\frac{G_a(\omega_0)}{\omega_0 C_T} = 2 \cdot 871 N |\Delta v/v|. \quad (24)$$

III. RECEIVING TRANSDUCER

A. Reciprocity: Reception by a Short-Circuited Transducer

In this section the process of reception by a transducer is analyzed in terms of the quasi-static approach. In order to do this, it is necessary first to rewrite some of the above equations in a different form. We consider the *electrostatic* relationship between charge $\sigma(x)$ and potential $\phi(x)$, obtained from (1)-(3), with $G_{SAW}(x)$ set to zero:

$$\begin{aligned} \phi(x) &= G_e(x) * \sigma(x) \\ &= [-\ln |x| / (\pi \epsilon_s(\infty))] * \sigma(x). \end{aligned} \quad (25)$$

For a transducer launching surface waves, this takes the form of (7). If $\phi(x)$ is known on the electrodes, a solution for $\sigma(x)$ can be found. However, $\phi(x)$ is not known *a priori* in the interelectrode gaps, and it is necessary to use the additional boundary condition that $\sigma(x) = 0$ in the gaps.

Here we use a formulation for $\sigma(x)$ based on the method used by Milsom *et al.* [13]. Define a set of N points x_j which exist only at the electrode locations and are equally spaced on the electrodes with spacing Δx . Fig. 2 shows this schematically. Then (25) can be written as

$$\phi(x_i) = \lim_{\Delta x \rightarrow 0} \sum_{j=1}^N A_{ij} \sigma(x_j) \quad (26)$$

with

$$\begin{aligned} A_{ij} &= \frac{1}{2} \Delta x [G_e(x_i - x_j + \frac{1}{2} \Delta x) + G_e(x_i - x_j - \frac{1}{2} \Delta x)] \\ &= -\frac{\Delta x}{2\pi \epsilon_s(\infty)} \cdot \ln \left| (x_i - x_j)^2 - \frac{1}{4} (\Delta x)^2 \right|. \end{aligned} \quad (27)$$

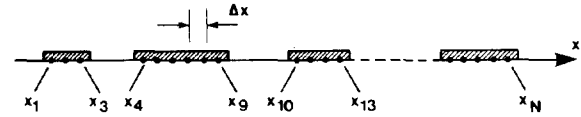


Fig. 2. Locations of the points x_j (schematic).

In (26) the limit is taken such that $N \rightarrow \infty$ as $\Delta x \rightarrow 0$, with $N \Delta x$ constant. A simpler choice for the matrix elements A_{ij} would be $\Delta x \cdot G_e(x_i - x_j)$, but this is not admissible here because it gives $A_{ii} = \infty$ when Δx is finite. Note that (26) implies that $\sigma(x) = 0$ in the gaps, because it involves values of $\sigma(x)$ only on the electrodes.

In the following analysis, the relations will be written for finite N and Δx , with the limit taken later. Thus (26) is written as

$$\phi(x_i) = \sum_{j=1}^N A_{ij} \sigma(x_j). \quad (28)$$

Defining a matrix B_{ij} as the reciprocal of the matrix A_{ij} , the charge density can be written in terms of the electrode potentials, without requiring the values of $\phi(x)$ in the gaps:

$$\sigma(x_i) = \sum_{j=1}^N B_{ij} \phi(x_j). \quad (29)$$

If a unit potential is applied across the transducer, then $\phi(x_j) = \frac{1}{2} P(x_j)$ where $P(x)$ is the polarity function defined in (17). The electrostatic charge density on the electrodes is then $\sigma_e(x)$ as defined in (7), so that

$$\sigma_e(x_i) = \frac{1}{2} \sum_j B_{ij} P(x_j). \quad (30)$$

Now consider the transducer to be short-circuited, with a surface wave incident from the left as in Fig. 1. The width of the incident beam is taken to be equal to the transducer aperture W , and its amplitude and phase are uniform across this width. The potential accompanying the incident wave is taken to be

$$\phi_I(x, \omega) = \phi_{I1}(\omega) \cdot \exp[-jk_0(x + \frac{1}{2}L)] \quad (31)$$

where $\phi_{I1}(\omega)$ is the potential at port 1, i.e., at $x = -\frac{1}{2}L$. Since the transducer is shorted, the electrodes are all at the same potential, and a charge distribution must exist on the electrodes in order to satisfy this condition. To evaluate the charge distribution, the electrode potentials can be taken to be zero, since a uniform potential applied to all electrodes gives rise to no charges. Thus the charge distribution must be such that, in the absence of an incident SAW, it would generate a potential distribution $-\phi_I(x_j, \omega)$ at the electrode locations. To solve for the charge distribution, the quasi-static approximation is used, i.e., the relation between charge and potential within the transducer is given by the electrostatic Green's function. Using (29) and (31), the charge distribution is

$$\sigma_R(x_i, \omega) = -\phi_{I1}(\omega) \exp(-\frac{1}{2}jk_0L) \sum_j B_{ij} \exp(-jk_0x_j). \quad (32)$$

Using the polarity function $P(x)$ defined in (17), the current generated in the short circuit external to the transducer is

$$I_{sc} = \frac{1}{2} j\omega W \sum_i P(x_i) \cdot \sigma_R(x_i, \omega) \cdot \Delta x. \quad (33)$$

Substituting from (32) and rearranging the order of summation gives

$$I_{sc} = -\frac{1}{2} j\omega W \cdot \phi_{I1}(\omega) \cdot e^{-1/2jk_0L} \sum_j \left[\sum_i P(x_i) B_{ij} \right] \cdot e^{-jk_0x_j} \cdot \Delta x. \quad (34)$$

Here the matrix B_{ij} is symmetrical. This follows because (27) shows A_{ij} to be symmetrical, so that the reciprocal matrix B_{ij} must also be symmetrical. Comparison with (30) then shows that the term in square brackets in (34) can be identified as $2\bar{\sigma}_e(x_j)$. In addition, the summation over j in (34) becomes an integral in the limit $\Delta x \rightarrow 0$, and since $\sigma_e(x)$ is zero except when x is on an electrode the integral can be seen to be the Fourier transform. Hence, in the limit $\Delta x \rightarrow 0$, (34) becomes

$$I_{sc} = -j\omega W \cdot \phi_{I1}(\omega) \cdot \exp(-\frac{1}{2}jk_0L) \cdot \bar{\sigma}_e(k_0). \quad (35)$$

Comparison with (13) shows that a reciprocity relation can be written in the form

$$\left[\frac{I_{sc}}{\phi_{I1}} \right]_{\text{receive}} = -\frac{\omega W}{|G_s|} \left[\frac{\phi_{s1}}{V_T} \right]_{\text{launch}} \quad (36)$$

This relation has been derived for the quasi-static case. It can however be shown to be valid for the more general case in which the surface-wave Green's function $G_{SAW}(x)$ is included. The argument is very similar. For the general case, $\phi(x)$ is related to $\sigma(x)$ by (1), with the Green's function given by (2). Instead of (28) we have

$$\phi(x_i) = \sum_j A'_{ij} \sigma(x_j, \omega) \quad (37)$$

where A'_{ij} is defined as in (27), but with $G_e(x) + G_{SAW}(x)$ replacing $G_e(x)$. For a unit potential applied across the transducer, the charge density is

$$\sigma_L(x_i, \omega) = \frac{1}{2} \sum_j B'_{ij} P(x_j) \quad (38)$$

which is similar to (30), with B'_{ij} the reciprocal of the matrix A'_{ij} . These matrices are complex and frequency-dependent, but it is readily shown that A'_{ij} , and therefore B'_{ij} , is symmetrical. For a receiving transducer, with the incident wave given by (31), the argument proceeds along the same lines, with B'_{ij} replacing B_{ij} and $\sigma_L(x, \omega)$ replacing $\sigma_e(x)$. Using $B'_{ji} = B'_{ij}$, the short-circuit current is found to be

$$I'_{sc} = -j\omega W \cdot \phi_{I1} \cdot e^{-1/2jk_0L} \int_{-\infty}^{\infty} \sigma_L(x, \omega) e^{-jk_0x} dx \quad (39)$$

instead of (35). Comparison with (5) then shows that the reciprocity relation of (36) is still valid. Thus the reciprocity relation applies even if electrode interactions are present.

B. Scattering Coefficients

The surface wave amplitudes generated by a transducer are given by (5) and (6) in terms of the charge density on the elec-

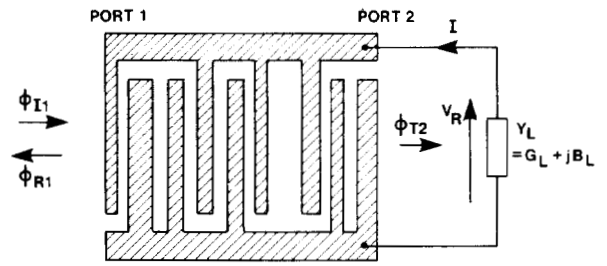


Fig. 3. Receiving transducer with arbitrary load admittance.

trodes. For a shorted transducer receiving surface waves, the charge density is given, in the quasi-static approximation, by (32). This charge density will therefore, in general, give rise to a reflected wave and a contribution to the transmitted wave, in addition to the incident wave given by (31). The amplitudes of these waves cannot be calculated in a *straight-forward* way from (32). However, it is known that it is often a good approximation to assume that the reflection coefficient of a shorted transducer is zero. For example, the well-known crossed-field model predicts this [2], [18]. For some cases, notably the single-electrode structure at its center frequency, the approximation is known to be invalid [19], but such cases are usually avoided in practical devices. It is therefore assumed here that the reflection coefficient for a shorted transducer is small enough to be ignored.

In general, the receiving transducer is taken to be connected to an arbitrary load, with admittance Y_L , as in Fig. 3. The transducer will therefore develop a voltage across its bus-bars, denoted V_R , and given by

$$V_R = -I_{sc}/(Y_T + Y_L) \quad (40)$$

where Y_T is the transducer admittance as given by (15). The charge density is now given by the short-circuit term (32) plus an additional term corresponding to the voltage V_R across the bus-bars. From the analysis of Section II-B, the additional term is, in the quasi-static case, simply $V_R \cdot \sigma_e(x)$. The waves radiated due to this term are given by (13) and (14), with V_T replaced by V_R . Denoting the reflected wave amplitude at port 1 by $\phi_{R1}(\omega)$, and using (35) and (40), we have

$$r_1 \equiv \frac{\phi_{R1}}{\phi_{I1}} = \frac{\omega W G_s}{Y_L + Y_T} [\bar{\sigma}_e(k_0)]^2 \cdot e^{-jk_0L} \quad (41)$$

where r_1 is the reflection coefficient at port 1.

To calculate the transmitted wave amplitude, it is again assumed that the charge density for the short-circuit case does not give any significant contribution to the radiated wave. For a finite load admittance, the quasi-static charge density $V_R \cdot \sigma_e(x)$ gives rise to a wave radiated to the right, with amplitude denoted by $\phi_{s2}(\omega)$ at port 2. This amplitude is given by (14), with V_R replacing V_T . Using (40) and (35) gives

$$\phi_{s2}(\omega) = -\phi_{I1}(\omega) \cdot G_a \cdot \exp(-jk_0L)/(Y_L + Y_T) \quad (42)$$

where (19) for G_a has also been used. The total wave amplitude, ϕ_{T2} , includes also the original incident wave, which at port 2 has amplitude $\phi_{I1}(\omega) \cdot \exp(-jk_0L)$. The transmis-

sion coefficient is thus given by

$$t_{12} = \frac{\phi_{T2}}{\phi_{T1}} \equiv \left[1 - \frac{G_a}{Y_L + Y_T} \right] \cdot e^{-jk_0 L}. \quad (43)$$

For a wave incident at port 2 instead of port 1, the reflection coefficient r_2 is the same as (41), but with $\bar{\sigma}_e^*(k_0)$ replacing $\bar{\sigma}_e(k_0)$. The transmission coefficient t_{21} is the same as t_{12} .

The power ratios for reflection and transmission of an incident wave are given by

$$|r_1|^2 = |r_2|^2 = G_a^2 / |Y_L + Y_T|^2 \quad (44)$$

$$|t_{12}|^2 = |t_{21}|^2 = 1 - \frac{G_a^2 + 2G_a \cdot G_L}{|Y_L + Y_T|^2} \quad (45)$$

where G_L is the real part of the load admittance Y_L . The reflection coefficient of (44) is obtained from (41) and (19).

For a wave incident on port 1, the power delivered to the load is $P_L = \frac{1}{2} |V_R|^2 \cdot G_L$. The power flow in the incident wave P_s is given by (18), with ϕ_{T1} replacing ϕ_s . Defining a power conversion coefficient as C_1 and using (35), (40), and (19), gives

$$C_1 = P_L / P_s = \frac{2G_a \cdot G_L}{|Y_L + Y_T|^2}. \quad (46)$$

For waves incident at port 2, the conversion coefficient C_2 is the same as C_1 . For waves incident on either acoustic port, it is seen that the sum of the power coefficients for reflection, transmission, and conversion is unity, as expected.

It should be noted that the conversion coefficient C_1 refers to the total power P_L dissipated in the load admittance Y_L . In general, a matching network is connected between the transducer and the actual load resistance, and Y_L must then be taken as the admittance seen looking into the matching network. Thus if the network has ohmic losses, the power P_L will be greater than the power delivered to the actual load. If the transducer itself has ohmic losses, these can be represented to first order by an external resistance, which must be included in the load admittance Y_L . Thus in this case P_L will include the ohmic power loss in the transducer itself.

The scattering coefficients given above are very general in that they are valid for almost arbitrary transducer geometries, taking full account of electrostatic effects which are included in the function $\sigma_e(x)$. The relations are also valid for arbitrary frequency. They are consistent with familiar properties which have previously been derived by the network approach. For example, a shorted transducer ($Y_L = \infty$) gives zero reflection coefficient. For a transducer tuned with a lossless inductor, $Y_L + Y_T = G_a$, and the power reflection coefficient is unity. If the transducer is matched at any frequency, then $Y_L + Y_T = 2G_a$, the power reflection and transmission coefficients are both $\frac{1}{4}$, and the conversion coefficient is $\frac{1}{2}$.

C. Scattering Matrix

The above equations are readily expressed as a scattering matrix relating the amplitudes of incident and radiated surface waves to the transducer voltage and current. For convenience, we define a surface wave amplitude $A(x, \omega)$ which is propor-

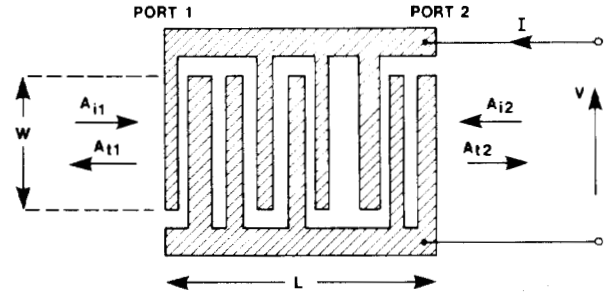


Fig. 4. Parameters defined at the two acoustic ports and the electrical port, related by the scattering matrix.

tional to the potential $\phi_s(x, \omega)$ accompanying the wave but is more directly related to the power density. The definition taken is

$$A(x, \omega) = \frac{1}{2} [\omega / |G_s|]^{1/2} \cdot \phi_s(x, \omega). \quad (47)$$

For a surface-wave beam of width W , the power flow is $W \cdot |A|^2$, by (18), so that $|A|^2$ is the power density. The phase of $A(x, \omega)$ is the same as that of the surface wave potential $\phi_s(x, \omega)$.

The amplitudes of waves incident on the transducer are denoted A_i and for waves leaving the transducer A_t . Additional subscripts 1 or 2 denote the ports, and the amplitudes are evaluated at these ports, i.e., at $x = \pm \frac{1}{2}L$. Thus A_{i1} , for example, is the amplitude of a wave incident on port 1, evaluated at $x = -\frac{1}{2}L$. The transducer current and voltage, I and V , are defined as in Fig. 4. We also define a function $E(\omega)$ such that, when a voltage V_T is applied to the transducer and no surface waves are incident, the SAW amplitude radiated at port 1 is given by

$$A_{t1} = jV_T \cdot E(\omega) \exp(-\frac{1}{2}jk_0L). \quad (48)$$

Using (13) and (47), this function is

$$E(\omega) = \frac{1}{2} [\omega \cdot |G_s|]^{1/2} \cdot \bar{\sigma}_e(k_0). \quad (49)$$

Note that $E(\omega)$ is real if the transducer geometry is symmetrical. From (19), it is related to $G_a(\omega)$ by

$$|E(\omega)|^2 = \frac{1}{4} G_a(\omega) / W. \quad (50)$$

With these definitions, the scattering matrix can be written

$$\begin{pmatrix} A_{t1} \\ A_{t2} \\ I/(4W) \end{pmatrix} = \begin{pmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \cdot \begin{pmatrix} A_{i1} \\ A_{i2} \\ V \end{pmatrix} \quad (51)$$

where

$$\begin{aligned} S_{12} &= S_{21} = \exp(-jk_0L) \\ S_{13} &= -S_{31} = jE(\omega) \exp(-\frac{1}{2}jk_0L) \\ S_{23} &= -S_{32} = jE^*(\omega) \exp(-\frac{1}{2}jk_0L) \\ S_{33} &= Y_T/(4W). \end{aligned}$$

This matrix is consistent with all the scattering parameters derived in Section III-B and can also be used to analyze more complex problems such as the transducer response when surface waves are incident on both acoustic ports.

IV. HIGHER ORDER ANALYSIS FOR TRANSDUCER ADMITTANCE

A. Basic Method

As pointed out in Section II-C it is not possible to deduce the correct transducer admittance by calculating the current using the quasi-static charge distribution, though the parallel conductance $G_a(\omega)$ can be obtained by considering the SAW power flow, giving (19). Here we derive a higher order approximation for the charge density, which leads to the same formula for $G_a(\omega)$ and also gives a formula for the susceptance $B_a(\omega)$.

Consider a transducer with unit voltage applied across it, so that $V_T = 1$ in Fig. 1. From (1) and (2), the potential and charge density are related by

$$\phi(x, \omega) = [G_e(x) + G_{\text{SAW}}(x)] * \sigma(x, \omega). \quad (52)$$

This relation is exact, apart from the neglect of bulk waves. For the quasi-static theory, $\sigma(x, \omega)$ was approximated by the function $\sigma_e(x)$, which is the electrostatic charge density obtained by ignoring $G_{\text{SAW}}(x)$ in (52). This function is the solution of

$$\phi'(x) = G_e(x) * \sigma_e(x) \quad (53)$$

where $\phi'(x)$ and $\phi(x, \omega)$ will have the same values on the electrodes but will in general be different in the gaps. The total charge density in (52) is now written

$$\sigma(x, \omega) = \sigma_e(x) + \sigma_a(x, \omega) \quad (54)$$

where $\sigma_a(x, \omega)$ can be regarded as the additional charge density due to SAW propagation and, unlike $\sigma_e(x)$, will be frequency-dependent. Here we consider an approximate analysis for $\sigma_a(x, \omega)$ and express it in terms of $\sigma_e(x)$. Substituting (54) into (52) gives four terms on the right side. The term $G_{\text{SAW}}(x) * \sigma_a(x, \omega)$ is small, however, since $G_{\text{SAW}}(x)$ and $\sigma_a(x, \omega)$ are considered to be small in relation to $G_e(x)$ and $\sigma_e(x)$, respectively. Thus (52) becomes

$$\begin{aligned} \phi(x, \omega) \approx & G_e(x) * \sigma_e(x) + G_{\text{SAW}}(x) * \sigma_e(x) \\ & + G_e(x) * \sigma_a(x, \omega). \end{aligned} \quad (55)$$

As in Section III-A, we consider a set of points x_j which exist only on the electrodes. Then since the potentials in (55) and (53) are the same on the electrodes, we have

$$\phi(x_j, \omega) = [G_e(x) * \sigma_e(x)]_{x_j}. \quad (56)$$

The remaining terms in (55) are therefore equal and opposite at points x_j . The second term can be identified as a potential arising due to the presence of SAW propagation. For convenience, this is denoted $\phi_a(x, \omega)$, so that

$$\phi_a(x, \omega) = G_{\text{SAW}}(x) * \sigma_e(x). \quad (57)$$

Thus, from (56) and (55), $\sigma_a(x, \omega)$ is the solution of

$$[G_e(x) * \sigma_a(x, \omega)]_{x_j} = -\phi_a(x_j, \omega). \quad (58)$$

From the analysis of Section III-A, the solution of this equation can be written

$$\sigma_a(x_i, \omega) = -\sum_j B_{ij} \phi_a(x_j, \omega) \quad (59)$$

where B_{ij} is the matrix relating electrostatic charge to potential, as in (29).

It is thus necessary to obtain $\phi_a(x, \omega)$ from (57). In principle, (59) must then be used for $\sigma_a(x, \omega)$, which is then added to $\sigma_e(x)$ to give the total charge density, as in (54). The total current can then be obtained in the usual way.

To evaluate $\phi_a(x, \omega)$, the surface wave Green's function $G_{\text{SAW}}(x)$ of (3) is substituted into (57), giving

$$\begin{aligned} \phi_a(x, \omega) = & -jG_s \int_{-\infty}^{\infty} \sigma_e(x') \cdot \exp(-jk_0|x-x'|) dx' \\ = & -jG_s \left\{ \int_{-\infty}^x \sigma_e(x') \exp[-jk_0(x-x')] dx' \right. \\ & \left. + \int_x^{\infty} \sigma_e(x') \exp[jk_0(x-x')] dx' \right\}. \end{aligned} \quad (60)$$

Thus $\phi_a(x, \omega)$ is essentially the Fourier transform of the static charge density to one side of x , plus the inverse transform of the charge density on the other side. By using standard properties of Fourier transforms, it is possible to express (60) in terms of $\bar{\sigma}_e(k)$, the Fourier transform of $\sigma_e(x)$. The steps necessary for this are given in Appendix B. The result is

$$\begin{aligned} \phi_a(x, \omega) = & -jG_s e^{-jk_0 x} \left\{ \frac{1}{2} \bar{\sigma}_e^*(k_0) - \frac{j}{2\pi} [\bar{\sigma}_e^*(k_0) * (e^{jk_0 x}/k_0)] \right\} \\ & -jG_s e^{jk_0 x} \left\{ \frac{1}{2} \bar{\sigma}_e(k_0) - \frac{j}{2\pi} [\bar{\sigma}_e(k_0) * (e^{-jk_0 x}/k_0)] \right\}. \end{aligned} \quad (61)$$

B. Evaluation of Capacitance and Conductance

The total transducer current is given by

$$I = \frac{1}{2} j\omega W \sum_i P(x_i) \cdot [\sigma_e(x_i) + \sigma_a(x_i, \omega)] \Delta x \quad (62)$$

where $P(x)$ is the polarity function defined in (17), and the term in square brackets is the total charge density $\sigma(x, \omega)$. This current will be written $I = I_e + I_a$, where I_e and I_a are contributions arising from $\sigma_e(x)$ and $\sigma_a(x, \omega)$, respectively. Since the applied voltage is unity, the transducer admittance is simply

$$Y_T = I_e + I_a. \quad (63)$$

Comparison with (16) shows that, in the limit $\Delta x \rightarrow 0$, the term I_e simply gives the capacitive part of the admittance:

$$I_e = j\omega C_T. \quad (64)$$

Using (59) for $\sigma_a(x_i, \omega)$, the remaining current contribution is

$$I_a = -\frac{1}{2} j\omega W \sum_i P(x_i) \sum_j B_{ij} \cdot \phi_a(x_j, \omega) \Delta x. \quad (65)$$

If the summations here are reordered and use is made of the fact that B_{ij} is symmetrical, the summation over i can be eliminated by using (30) for $\sigma_e(x_j)$. This gives

$$I_a = -j\omega W \sum_j \sigma_e(x_j) \cdot \phi_a(x_j, \omega) \Delta x. \quad (66)$$

Now, (61) gives $\phi_a(x, \omega)$ as a sum of four terms. We write I_a as a sum of two components, $I_a = I_{a1} + I_{a2}$, where I_{a1} arises from the first and third terms, and I_{a2} arises from the second and fourth terms. It will be shown that I_{a1} gives the conductance $G_a(\omega)$, while I_{a2} gives the susceptance $B_a(\omega)$. Substituting the first and third terms of (61) into (66) gives

$$I_{a1} = -\frac{1}{2} \omega W G_s \Delta x \left[\bar{\sigma}_e^*(k_0) \sum_j \sigma_e(x_j) e^{-jk_0 x_j} + \bar{\sigma}_e(k_0) \sum_j \sigma_e(x_j) e^{jk_0 x_j} \right]. \quad (67)$$

Here the first summation becomes an integral when $\Delta x \rightarrow 0$, and since $\sigma_e(x)$ is zero in the gaps the integral can be taken over all x , giving the Fourier transform $\bar{\sigma}_e(k_0)$ of $\sigma_e(x)$. Similarly, the second summation gives $\bar{\sigma}_e(-k_0)$, which can be written as $\bar{\sigma}_e^*(k_0)$ because $\sigma_e(x)$ is real. Hence

$$I_{a1} = -\omega W G_s |\bar{\sigma}_e(k_0)|^2. \quad (68)$$

This is real and positive. It is shown below that the remaining current contribution I_{a2} is imaginary. Hence I_{a1} is the only real contribution to the current, and therefore gives the parallel conductance $G_a(\omega)$ as

$$G_a(\omega) = -\omega W G_s |\bar{\sigma}_e(k_0)|^2. \quad (69)$$

This agrees with the formula obtained from the power flow of the radiated surface waves (19).

C. Evaluation of Susceptance

The remaining terms in (61), the second and fourth terms, must be substituted into (66) to give the remaining current contribution I_{a2} . Consider initially the second term only of (61). Substituting into (66), this gives a current contribution I'_{a2} , say, given by

$$I'_{a2} = \frac{j\omega W G_s \Delta x}{2\pi} \sum_j \sigma_e(x_j) e^{-jk_0 x_j} [\bar{\sigma}_e^*(k_0) * (e^{jk_0 x_j} / k_0)]. \quad (70)$$

Writing out the convolution as an integral, and reordering the integral and summation, gives

$$I'_{a2} = \frac{j\omega W G_s}{2\pi} \int_{-\infty}^{\infty} \left[\sum_j \sigma_e(x_j) e^{-jk_0 x_j} \Delta x \right] \frac{\bar{\sigma}_e^*(k'_0)}{k_0 - k'_0} dk'_0. \quad (71)$$

Here the term in square brackets can be identified as $\bar{\sigma}_e(k'_0)$ in the limit $\Delta x \rightarrow 0$, using the same argument as applied to the similar term in (67). Thus (71) gives

$$I'_{a2} = \frac{j\omega W G_s}{2\pi} [|\bar{\sigma}_e(k_0)|^2 * (1/k_0)]. \quad (72)$$

Finally, it is necessary to substitute the fourth term of (61) into (66) to give the remaining contribution to I_{a2} . The method is almost identical to the above analysis for I'_{a2} , except that it is necessary to use the fact that $\bar{\sigma}_e(-k) = \bar{\sigma}_e^*(k)$, which follows because $\sigma_e(x)$ is real. The result is the same as (72), so the total contribution I_{a2} is simply $2I'_{a2}$. Since this

is imaginary, I_{a2} can be identified with $jB_a(\omega)$, so that

$$B_a(\omega) = I_{a2} / j = \frac{\omega W G_s}{\pi} \left[|\bar{\sigma}_e(k_0)|^2 * \frac{1}{k_0} \right]. \quad (73)$$

We have thus shown that the transducer current I of (62) gives an admittance $Y_T = G_a + jB_a + j\omega C_T$, with G_a and B_a given by (69) and (73), respectively, and C_T given by (16).

D. Consistency with Quasi-Static Approach

In Sections IV-A-C, it has been necessary to use a higher-order approximation to the transducer charge density in order to derive the admittance from the transducer current. In Section II, on the other hand, a first-order approximation was used, taking the charge density as $\sigma_e(x)$, but nevertheless the transducer conductance $G_a(\omega)$ obtained by analyzing the SAW power flow gave the same result. These results are at first sight inconsistent.

The reason for this can be seen by considering (55), which gives the potential $\phi(x, \omega)$ for the higher order approximation. In the above analysis, we have used this equation to solve for charges and potentials within the transducer. The equation is however also valid for points outside the transducer. At such points the potential $\phi(x, \omega)$ has three terms, but only one of them, $G_{SAW}(x) * \sigma_e(x)$, gives a sinusoidal potential corresponding to propagating waves. We can therefore identify this term with the potential $\phi_s(x, \omega)$ accompanying the surface waves in the regions outside the transducer, so that

$$\phi_s(x, \omega) = G_{SAW}(x) * \sigma_e(x) \quad (74)$$

in the higher-order approximation. The other two terms in (55) give contributions to the potential which depend on the electrostatic Green's function $G_e(x)$, and these will decay with distance away from the transducer.

The form for $\phi_s(x, \omega)$ given by (74) is identical to (4), which was used for the first order quasi-static approximation. The two approximations therefore give identical results for the surface wave amplitudes launched by the transducer. Calculation of $G_a(\omega)$ from the SAW powers gives the same result in both approximations and the apparent inconsistency is thus removed. It also follows that, in the higher order approximation, the conductance can be obtained either from the SAW powers or from the transducer current, and the two methods give the same result.

E. Relation between Conductance and Susceptance

The relation between $G_a(\omega)$ and $B_a(\omega)$ is essentially a convolution in the frequency domain. This involves negative frequencies, and since the above analysis assumes $\omega \geq 0$, the formulae are rewritten so as to be valid for both positive and negative frequencies. Since $G_a(\omega)$ must be symmetrical with ω , and $B_a(\omega)$ antisymmetrical, (69) and (73) become

$$G_a(\omega) = |\omega G_s| \cdot W \cdot |\bar{\sigma}_e(\omega/v_0)|^2 \quad (75)$$

$$B_a(\omega) = -\frac{|\omega G_s|}{\pi} \cdot W \cdot \left[|\bar{\sigma}_e(\omega/v_0)|^2 * \frac{1}{\omega} \right]. \quad (76)$$

Here the substitution $k_0 = \omega/v_0$ has also been made. These

quantities are thus related by

$$\begin{aligned} B_a(\omega) &= -\frac{|\omega|}{\pi} \left[\frac{G_a(\omega)}{|\omega|} * \frac{1}{\omega} \right] \\ &= -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{|\omega| \cdot G_a(\omega')}{|\omega'| \cdot (\omega - \omega')} d\omega'. \end{aligned} \quad (77)$$

It can be assumed to a high degree of accuracy that, for positive ω , the value of $B_a(\omega)$ depends only on the values of $G_a(\omega')$ at positive ω' . In addition, for negative ω , $B_a(\omega)$ depends only on values of $G_a(\omega')$ at negative ω' . Then for either positive or negative ω , (77) can be written

$$\begin{aligned} B_a(\omega) &= -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\omega G_a(\omega')}{\omega'(\omega - \omega')} d\omega' \\ &= -\frac{1}{\pi} \int_{-\infty}^{\infty} \left[\frac{G_a(\omega')}{\omega - \omega'} + \frac{G_a(\omega')}{\omega'} \right] d\omega' \end{aligned} \quad (78)$$

$$= -\frac{1}{\pi} G_a(\omega) * \frac{1}{\omega} \quad (79)$$

where the second term in (78) has disappeared because $G_a(\omega')$ is symmetrical with respect to ω' . Hence $B_a(\omega)$ is related to $G_a(\omega)$ by the Hilbert transform.

V. CONCLUSION

It has been shown that, by assuming that bulk wave excitation and electrode interactions are negligible, a first-order approximation of the Green's function theory leads to simple analytic expressions for the transducer properties. Formulae were given for the amplitude of surface waves launched, the transducer conductance and susceptance, and the scattering coefficients for a transducer receiving surface waves. The formulae are all expressed in terms of $\sigma_e(x)$ or its Fourier transform $\bar{\sigma}_e(k)$, where $\sigma_e(x)$ is the charge density on the electrodes when unit potential is applied across the bus-bars, obtained by a purely electrostatic analysis ignoring piezoelectricity. The susceptance $B_a(\omega)$ was obtained by a second-order approximation, and it was shown that the second-order approximation gives the same results as the first for the amplitudes of waves launched. The transducer conductance $G_a(\omega)$ can be obtained from the surface wave power flow in either approximation, or from analysis of the transducer current in the second-order approximation; the three cases give the same result. The susceptance was shown to be the Hilbert transform of the conductance. A reciprocity relation was shown to be valid even if electrode interactions are present. All of the above relationships are valid for almost arbitrary transducer geometries, as defined in Section I, the main restriction being that the transducer is not apodized. The generalization to apodized transducers is however straightforward, using Tancrell and Holland's concept of dividing an apodized transducer into a set of unapodized channels [3].

The formulae given here take full account of electrostatic effects, which are included in the function $\sigma_e(x)$. As mentioned in the Introduction, there are many cases where these effects are of practical significance and must therefore be allowed for accurately. The solution of the electrostatic problem has been considered by a number of authors and used to obtain the

surface wave amplitude. For other transducer properties, Smith and Peddler [9] used a network approach based on the electrostatic solution, at the expense of some complexity. Wagers [12] has shown how the normal mode theory can be used to relate the transducer admittance to the electrostatic solution, but does not describe other transducer properties. The relationships given here are very simple and cover a wide range of transducer properties. The derivation also gives physical insight into the transducer behavior and is more rigorous than the network approach.

The validity of the theory depends on the choice of substrate material and orientation. It is assumed that the substrate gives piezoelectric coupling only to a nonleaky SAW mode, thus excluding in principle substrates which couple to other modes. However, if the coupling to bulk waves is weak enough the theory will be approximately valid, and for most SAW materials it is expected to give sufficient accuracy for practical purposes. The validity can be assessed by examining the Green's function $G(x)$ as described by Milsom *et al.* [13]. It has also been assumed that electrode interactions are negligible, and this is valid if the piezoelectric coupling is weak. The effect of electrode interactions also depends on the transducer geometry, however. For example, in a single-electrode transducer at the center frequency, interactions give rise to strong reflections. The double-electrode geometry suppresses these reflections [19], and in this case the main consequence of electrode interactions is effectively a change of SAW velocity within the transducer. It is therefore expected that the theory should be valid for substrates with strong piezoelectric coupling, such as *Y-Z* lithium niobate, provided the transducer structure is such as to suppress the reflections, and provided the velocity change is allowed for.

The electrostatic problem, which must be solved to obtain the function $\sigma_e(x)$, has not been considered here. For simpler cases, where good accuracy is not essential, the function can often be obtained readily from the analytic results of Engan [5], [6]. More generally, accurate results for arbitrary transducer geometries can be obtained by numerical techniques [7]-[9], [13], [21]. Alternatively, for most practical transducers the charge density can be expressed in terms of a more fundamental function using the superposition principle [15]-[17]. This enables accurate electrostatic solutions to be obtained very readily without iterative procedures and is applicable, for example, to transducers with arbitrary polarity sequences.

The theory presented here assumes that the transducer has two bus-bars, thus excluding three-phase and group-type unidirectional transducers. The theory is, however, readily generalized to include these cases.

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APPENDIX A RELATION BETWEEN G_s AND $\Delta v/v$

The general relation between potential $\phi(x, \omega)$ and charge density $\sigma(x, \omega)$ is, from (1)-(3),

$$\phi(x, \omega) = G(x) * \sigma(x, \omega) \quad (A1)$$

with

$$G(x) = -\ln |x| / [\pi \epsilon_s(\infty)] - jG_s \exp(-jk_0|x|). \quad (\text{A2})$$

It is assumed that there are no piezoelectrically coupled bulk modes and that for a free surface a nonleaky surface wave can propagate with velocity v_0 and wavenumber $k_0 = \omega/v_0$. Under these conditions, the above equations are exact. We also assume that a nonleaky surface wave mode exists, with velocity v_m , when the surface is metallized with a thin, infinitely conducting film. A solution giving this velocity should be obtainable from (A1) by setting $\phi(x, \omega) = 0$. The solution is found by taking the Fourier transform of (A1), which is

$$\bar{\phi}(k) = \bar{G}(k) \cdot \bar{\sigma}(k) \quad (\text{A3})$$

where $\bar{\phi}(k)$, $\bar{G}(k)$, and $\bar{\sigma}(k)$ are the Fourier transforms of $\phi(x)$, $G(x)$, and $\sigma(x)$, respectively, and the frequency argument has been omitted since it will remain constant throughout. On a metallized surface, $\phi(x) = 0$ for all x , and hence $\bar{\phi}(k) = 0$ for all k . Thus the solution is obtained by setting the right side of (A3) to zero for all k .

To evaluate $\bar{G}(k)$, we first find the transforms of the functions in (A2). The differential of $\ln |x|$ is x^{-1} , and from [20, p. 130], the Fourier transform of x^{-1} is $-j\pi \operatorname{sgn}(k)$. Since the transform of the differential of a function is jk times the transform of the function, the transform of the logarithm is given by

$$F[\ln |x|] = -\pi/|k| \quad (\text{A4})$$

where the notation $F[\cdot]$ indicates the Fourier transform.

The exponential in (A2) can be written

$$e^{-ja|x|} = e^{-jax} \cdot H(x) + e^{jax} \cdot H(-x) \quad (\text{A5})$$

where $H(x)$ is the Heaviside function [20] defined by

$$\begin{aligned} H(x) &= 1, & \text{for } x > 0 \\ &= 0, & \text{for } x < 0. \end{aligned} \quad (\text{A6})$$

Now

$$F[\exp(\pm jax)] = 2\pi\delta(k \mp a) \quad (\text{A7})$$

and from [20, p. 365]

$$F[H(\pm x)] = \pi\delta(k) \mp j/k. \quad (\text{A8})$$

To calculate the products in (A5), we also use the convolution theorem

$$F[f_1(x) \cdot f_2(x)] = \bar{f}_1(k) * \bar{f}_2(k) / (2\pi) \quad (\text{A9})$$

where $\bar{f}_1(k)$ and $\bar{f}_2(k)$ are the transforms of $f_1(x)$ and $f_2(x)$. From (A7)-(A9),

$$F[e^{-jax} \cdot H(x)] = \pi\delta(k+a) - j/(k+a) \quad (\text{A10})$$

and

$$F[e^{jax} \cdot H(-x)] = \pi\delta(k-a) + j/(k-a). \quad (\text{A11})$$

The transform of $\exp(-ja|x|)$ is the sum of these, by (A5). Using (A2), (A4), (A10), and (A11), the transform of $G(x)$ is

$$\begin{aligned} \bar{G}(k) &= [|k| \cdot \epsilon_s(\infty)]^{-1} - jG_s [\pi\delta(k+k_0) \\ &+ \pi\delta(k-k_0) - j/(k+k_0) + j/(k-k_0)]. \end{aligned} \quad (\text{A12})$$

For a metallized surface, we take $\sigma(x) = \exp(-jk_m x)$, where k_m is the wavenumber for this case. Its transform is $\bar{\sigma}(k) = 2\pi\delta(k+k_m)$ and this is zero except for $k = -k_m$. Thus to make the right side of (A3) zero for all k , we must have $\bar{G}(-k_m) = 0$. It can be assumed that $k_m \neq \pm k_0$, since metallization will change the velocity, and this eliminates the delta-functions in (A12). Putting $\bar{G}(-k_m) = 0$ gives

$$G_s = \frac{1}{2\epsilon_s(\infty)} \cdot \left(\frac{v_m}{v_0} - \frac{v_0}{v_m} \right) \approx \frac{1}{\epsilon_s(\infty)} \cdot \frac{v_m - v_0}{v_0} \quad (\text{A13})$$

where we have used $k_m > 0$ and replaced k_0 and k_m by using the free-surface and metallized velocities, given by $k_0 = \omega/v_0$ and $k_m = \omega/v_m$.

APPENDIX B

DERIVATION OF $\phi_a(x, \omega)$

In Section IV-B, the acoustic potential $\phi_a(x, \omega)$ is given as a pair of integrals, in (60). Here this is expressed in terms of $\bar{\sigma}_e(k_0)$. Using the Heaviside function $H(x)$, defined as in Appendix A, (60) becomes

$$\begin{aligned} \phi_a(x, \omega) &= -jG_s \left\{ e^{-jk_0 x} \int_{-\infty}^{\infty} e^{jk_0 x'} \sigma_e(x') \cdot H(x-x') dx' \right. \\ &\quad \left. + e^{jk_0 x} \int_{-\infty}^{\infty} e^{-jk_0 x'} \sigma_e(x') \cdot H(x'-x) dx' \right\}. \end{aligned} \quad (\text{B1})$$

The second integral here is a Fourier transform. The first integral can be evaluated by taking the Fourier transform and then taking the conjugate, because $\sigma_e(x') \cdot H(x-x')$ is real. Denoting the Fourier transform of any function $f(x)$ by $F[f(x)]$, which will be a function of k ,

$$\begin{aligned} \phi_a(x, \omega) &= -jG_s \{ e^{-jk_0 x} (F[\sigma_e(x') \cdot H(x-x')])^* \\ &\quad + e^{jk_0 x} \cdot F[\sigma_e(x') \cdot H(x'-x)] \}. \end{aligned} \quad (\text{B2})$$

Here the Fourier transforms are from the x' domain to the k domain and are to be evaluated at $k = k_0$. The transforms can be evaluated using relationships given in Appendix A; (A8) gives the transform of $H(x)$, and the convolution theorem of (A9) is used for the product. Using (A8) and the standard theorem for the transform of a function displaced in the x' -domain,

$$F[H(x-x')] = \pi\delta(k) + j \exp(-jkx)/k$$

and

$$F[H(x'-x)] = \pi\delta(k) - j \exp(-jkx)/k.$$

Using the convolution theorem of (A9),

$$(F[\sigma_e(x') \cdot H(x-x')])^* = \frac{1}{2} \bar{\sigma}_e^*(k) - \frac{j}{2\pi} \bar{\sigma}_e^*(k) * [\exp(jkx)/k]$$

and

$$F[\sigma_e(x') \cdot H(x'-x)] = \frac{1}{2} \bar{\sigma}_e(k) - \frac{j}{2\pi} \bar{\sigma}_e(k) * [\exp(-jkx)/k].$$

Evaluating these functions at $k = k_0$ and substituting into (B2) gives (61), which is the required result.

APPENDIX C

TRANSDUCERS WITH DISCONNECTED ELECTRODES

This appendix generalizes the theory to include transducers such as that shown in Fig. 5, in which some of the electrodes are not connected to either bus-bar. An example is the six-electrode transducer described by Engan [6]. It is shown here that all the results of the main paper are also valid for transducers with disconnected electrodes. As before, the results are expressed in terms of the electrostatic charge density $\sigma_e(x)$ which is produced when unit potential is applied across the bus-bars and its Fourier transform $\bar{\sigma}_e(k)$. Since the potentials of the disconnected electrodes are initially unknown, additional boundary conditions are required for evaluation of $\sigma_e(x)$, and these are that the net charge on each disconnected electrode is zero.

To analyze the transducer, it is necessary to generalize the electrostatic formulation described in Section III-A. As before, define a sequence of points $x_j, j = 1, 2, \dots, N$ which occur on the electrodes but not elsewhere. These points will occur on the disconnected electrodes as well as on the connected ones. The number of disconnected electrodes is M , so that $M = 2$, in Fig. 5, for example. Indices μ and ν indicate the disconnected electrodes, taking values $1, 2, \dots, M$.

We consider the *electrostatic* solution for charge density $\sigma(x)$, which must satisfy (25). If the electrode potentials are given by a known function $\phi(x_j)$, the solution is given by (29) in terms of the matrix B_{ij} . Here, however, the potentials of the disconnected electrodes are initially unknown, so a somewhat *different problem* is considered. Given a function $\phi(x_j)$, the transducer potential is taken to be $\phi(x_j)$ if x_j is on a connected electrode, and $[\phi(x_j) + V_\mu]$ if x_j is on disconnected electrode, μ . Thus, for each disconnected electrode, the potential differs from $\phi(x_j)$ by a constant. The constants V_μ are introduced in order to satisfy the condition that each disconnected electrode has zero net charge. By superposition, the total charge density $\sigma(x)$ is the sum of the charge density due to the potential $\phi(x_j)$ and the charge densities due to the potentials V_μ , so that

$$\sigma(x_i) = \sum_{j=1}^N B_{ij} \phi(x_j) + \sum_{\mu=1}^M \sum_m B_{im} V_\mu. \quad (C1)$$

Here the notation \sum_m^μ indicates that the sum is to be taken over those values of m for which x_m lies on disconnected electrode μ . On each disconnected electrode, the net charge is zero, giving

$$\sum_n \sigma(x_n) = 0, \quad \nu = 1, 2, \dots, M. \quad (C2)$$

Substitution of (C1) into (C2) gives M equations in the M unknowns V_μ , which can therefore be written as linear sums of the potentials $\phi(x_j)$. Hence the charge density $\sigma(x_i)$ can be written in terms of $\phi(x_j)$, and we define a matrix E_{ij} such that (C1) becomes

$$\sigma(x_i) = \sum_j [B_{ij} + E_{ij}] \phi(x_j). \quad (C3)$$

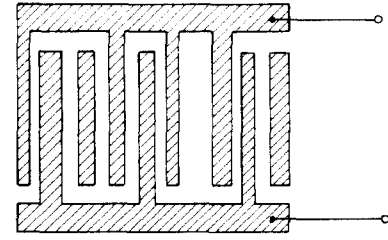


Fig. 5. Transducer with disconnected electrodes.

This equation expresses the charge density in terms of the potential $\phi(x_j)$, without explicitly involving the additional potentials V_μ on the disconnected electrodes.

From the above equations, it can be shown that E_{ij} is given by

$$E_{ij} = - \sum_{\mu} \sum_{\nu} K_{\mu\nu} \left(\sum_m^{\mu} B_{im} \right) \cdot \left(\sum_n^{\nu} B_{nj} \right) \quad (C4)$$

where $K_{\nu\mu}$ is the inverse of a matrix $C_{\nu\mu}$ defined by

$$C_{\nu\mu} = \sum_n^{\nu} \sum_m^{\mu} B_{nm}. \quad (C5)$$

It is readily shown that, since B_{ij} is symmetrical, E_{ij} will also be symmetrical.

Using (C3) for the charge density in place of (29), the theory for a transducer with disconnected electrodes follows the same arguments as in the main paper. The electrostatic charge density $\sigma_e(x)$ produced when unit potential is applied across the bus-bars is obtained by setting $\phi(x_j) = \pm \frac{1}{2} V$ in (C3) for the connected electrodes, with the sign corresponding to the electrode polarity. On each disconnected electrode $\phi(x_j)$ must be invariant, so that the total potential, which includes the V_μ , is constant. The value assigned to $\phi(x_j)$ is however irrelevant, as any change of this value is implicitly compensated by a change of V_μ . It is convenient to take the value of $\phi(x_j)$ as zero on the disconnected electrodes, and hence $\sigma_e(x)$ is given by

$$\sigma_e(x_i) = \frac{1}{2} \sum_j [B_{ij} + E_{ij}] \cdot P_1(x_j). \quad (C6)$$

Here $P_1(x)$ is a polarity function similar to the function $P(x)$ of (17), and is defined by $P_1(x) = \pm 1$ when x is on a connected electrode, with the sign given by the polarity of the electrode, and $P_1(x) = 0$ when x is on a disconnected electrode. The transducer static capacity C_T is given by (16), with $P_1(x)$ replacing $P(x)$ and (C6) for $\sigma_e(x)$.

For a short-circuited receiving transducer, the analysis is similar to that of Section III-A. The charge density is given by (C3), with $\phi(x_j)$ replaced by $-\phi_I(x, \omega)$ using (31), so that the potential is zero at the locations of the connected electrodes. The remaining analysis is the same, except that $P(x)$ is replaced by $P_1(x)$ and B_{ij} is replaced by $[B_{ij} + E_{ij}]$, which is symmetrical. The reciprocity relation of (36) is therefore still valid, both for the quasi-static case and for the more general case in which the surface-wave Green's function is included.

The higher-order analysis for the transducer admittance is almost identical to that of Section IV. The solution of (58) must be obtained by using (C3), so that $\sigma_a(x_i, \omega)$ is given by

(C3) with $\phi(x_j)$ replaced by $-\phi_a(x_j, \omega)$. The function $P_1(x)$ replaces $P(x)$. From (66) onwards, the analysis is the same.

In summary, all of the equations in the main paper giving transducer properties in terms of $\sigma_e(x)$, $\bar{\sigma}_e(k_0)$ or $G_a(\omega)$ are also valid for transducers with disconnected electrodes. The presence of disconnected electrodes is allowed for implicitly by defining $\sigma_e(x)$ as the electrostatic charge density produced when unit potential is applied across the transducer, with the potentials of disconnected electrodes allowed to float. The expression for the static capacity (16) is slightly modified in that a different polarity function is used.

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