V. SUMMARY AND CONCLUSION

A new theory for the p^+n SAW memory correlator operated in the parametric mode has been described and shown to be in good agreement with experimental data obtained with such a structure. A general formula for the stored charge in the diodes which takes into account the recombination of minority carriers in the neutral region has been given, thus filling a gap in our quantitative understanding of the role of minority carrier lifetimes in the operation of SAW memory correlators.

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Element Factor for Periodic Transducers

SUPRIYO DATTA AND BILL J. HUNSINGER

Abstract—**It was shown previously that the response of a periodic transducer with arbitrary voltages can be written as the product of an "element factor" and an "array factor." However, the element factor was obtained from numerical tables that take the effect of only two nearest neighbors into account. The element factor is here obtained analytically with the effects of all neighboring electrodes accounted for.**

A LARGE NUMBER of practical transducers are modeled as a periodic array of metallic strips with arbitrary impressed voltages. It has been shown [1], [2] that the response of such periodic transducers is written as the product of an "element factor" and an "array factor." The element factor was obtained in [1] using the numerical tables of [3] which take the effects of two neighboring electrodes into account. In this paper analytical expressions for the element factor are obtained for all frequencies and metallization ratios with the effects of all neighboring electrodes accounted for. The response of the periodic transducer (neglecting regeneration and reflection) is thus obtained analytically for all frequencies.

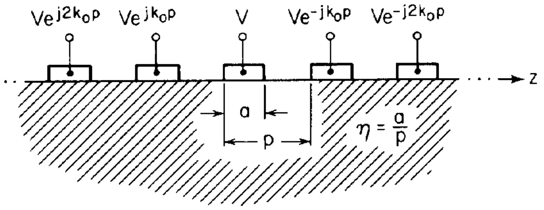

Fig. 1. A phased array transducer with a uniform phase progression of k_0p per period.

We first consider a phased array transducer (Fig. 1) in which the voltages on all the electrodes have the same magnitude V but the phase progresses uniformly along the array at the rate of k_0p per period where p is the period. The voltage on the nth electrode, V_n is written as

$$\begin{aligned} V_n &= V \exp(-jnk_0p) \\ &= V \exp(-jn2\pi s_0) \end{aligned} \tag{1}$$

where s_0 is a dimensionless variable related to k_0 by

$$s_0 = k_0p/2\pi. \tag{2}$$

It will be noticed from (1) that adding any integer to s_0 yields the same electrode voltages so that we may without loss of generality restrict the values of s_0 such that

$$0 < s_0 < 1. \tag{3}$$

The charge distribution in this transducer is known to be [4]

$$Q(\theta) = \frac{2V}{p}(\epsilon_p + \epsilon_0) \cdot \frac{\sin \pi s_0}{P_{s_0-1}(-\cos\Delta)} \sum_{n=-\infty}^{\infty} P_n(\cos\Delta) \cdot e^{-j(s_0+n)\theta} \tag{4}$$

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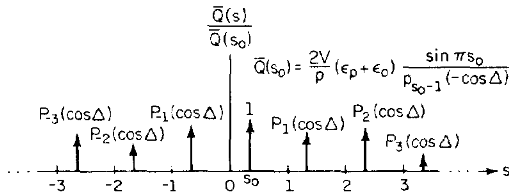

Fig. 2. Fourier transform of charge distribution for a phased array transducer.

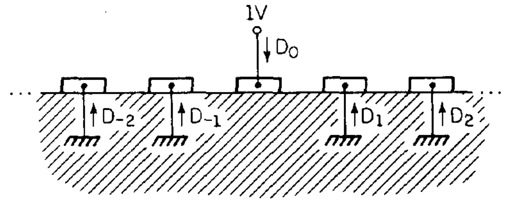

Fig. 3. Electrode currents in a single tap transducer.

where

- θ dimensionless variable related to z by $\theta = 2\pi z/p$;
- ϵ_p effective permittivity of the substrate;
- ϵ_0 permittivity of the medium above the surface;
- Δ $= \eta\pi$, η being the metallization ratio; and
- P_ν Legendre function of order ν (integer or noninteger).

From (4) we note that the Fourier transform $\bar{Q}(s)$ of $Q(\theta)$ has delta function components at all s given by $s = s_0 + n$, n being an integer:

$$\bar{Q}(s) = \bar{Q}(s_0) \cdot P_n(\cos\Delta) \tag{5a}$$

where $s = s_0 + n$ and

$$\bar{Q}(s_0) = \frac{2V}{p}(\epsilon_p + \epsilon_0)\frac{\sin \pi s_0}{P_{s_0-1}(-\cos\Delta)}. \tag{5b}$$

Here s is the usual spatial harmonic k scaled by $p/2\pi$ to make it dimensionless. Fig. 2 shows a plot of $\bar{Q}(s)$ against s for the phased array transducer with electrode voltages given by (1).

The electrode current at each strip has the same magnitude but with a phase progression along the array. The current D_n at the nth strip is written as

$$D_n = D_0 \exp(-js_0 2\pi n) \tag{6a}$$

where D_0 is obtained by integrating $j\omega Q(\theta)$ over the width of a strip

$$D_0 = j\omega W \int_{-\Delta}^{+\Delta} d\theta\, Q(\theta)$$

$$= j\omega W(\epsilon_p + \epsilon_0) \cdot 2V \sin \pi s_0 \frac{P_{s_0-1}(\cos\Delta)}{P_{s_0-1}(-\cos\Delta)}. \tag{6b}$$

W is the length of a strip in the direction perpendicular to the wave propagation.

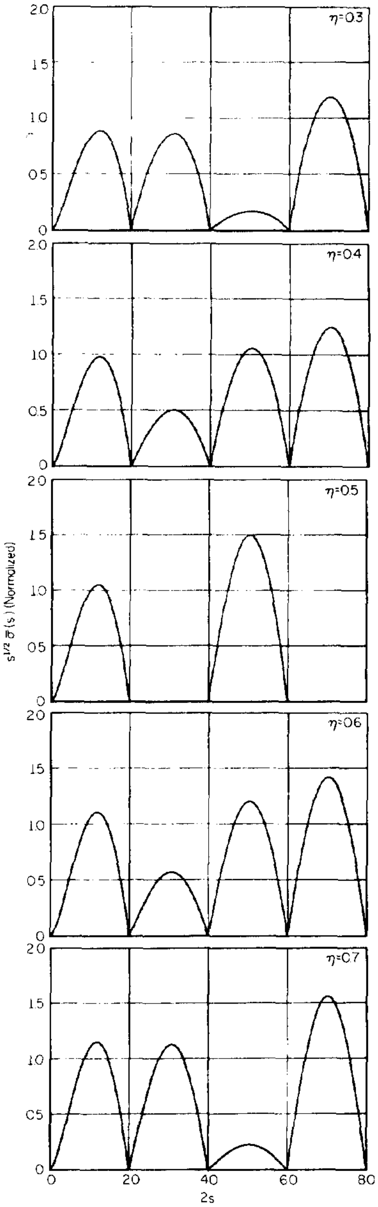

Fig. 4. Element factor for different metallization ratios.

Now, we wish to determine the charge distribution in a single tap transducer (Fig. 3) since this yields the element factor [1], [2]. It is easily seen that this is obtained by superposing the charge distributions in an infinite number of phased array transducers each with $V = 1$, but with s_0 varying uniformly from 0 to 1. The voltage on the nth electrode is then obtained from (1) as

$$V_n = \int_0^1 ds_0 \exp(-jn2\pi s_0)$$

$$= 1, \text{for } n = 0$$

$$= 0, \text{for } n \neq 0$$

which clearly describes the single tap structure in Fig. 3.

Hence the Fourier transform of the charge distribution in a single tap structure is a continuous function of s given by

$$\bar{\sigma}(s, \Delta) = \frac{2(\epsilon_p + \epsilon_0)}{p}\frac{\sin \pi s_0}{P_{s_0-1}(-\cos\Delta)} \cdot P_n(\cos\Delta) \tag{7}$$

where $s = s_0 + n$ such that $0 < s_0 < 1$ and n is an integer. Fig. 4 shows $s^{1/2}\,\bar{\sigma}(s, \Delta)$ plotted against $2s$, for different values of the metallization ratio. These agree reasonably with the numerical plots in [1, fig. 3] where $2s$ corresponds to f/f_0. The discrepancies are possibly because only two nearest neighbors were accounted for in the numerical method.

The capacitance of periodic transducers was obtained in [1] in terms of the electrode currents, D_n (Fig. 3) in the single tap transducer. These are readily obtained from (6) by integrating over s_0 from 0 to 1:

$$D_n = 2j\omega W(\epsilon_p + \epsilon_0) \cdot \int_0^1 \sin \pi s_0 \, \frac{P_{s_0 - 1}(\cos \Delta)}{P_{s_0 - 1}(-\cos \Delta)} \, e^{-jn2\pi s_0} \, ds_0. \tag{8}$$

In general the integral has to be obtained numerically, but for $\eta = 0.5$, $\cos \Delta = 0$; so that $P_{s_0 - 1}(\cos \Delta) = P_{s_0 - 1}(-\cos \Delta)$. D_n is then obtained analytically as

$$D_0 = j\omega W(\epsilon_p + \epsilon_0)\frac{4}{\pi} \tag{9a}$$

$$D_n = \frac{D_0}{1 - 4n^2}. \tag{9b}$$

It is seen that, while D_0 is positive, all other D_n are negative as expected. It may be shown that, as expected, the sum of all the electrode currents is zero:

$$\sum_{n=-\infty}^{\infty} D_n = 0.$$

CONCLUSION

In [1] the conductance and capacitance of a periodic transducer with arbitrary voltages were related to the charge distribution and the electrode currents in a single tap transducer. However, these quantities were obtained from numerical tables that take only two nearest neighbors into account. In this paper the charge distribution and the electrode currents in a single tap transducer are obtained analytically considering the effects of all the nearest neighbors, making the analysis more accurate and more convenient. The overall charge distribution in a periodic transducer is given by [1, eq. (5)] as the product (in k-space) or the convolution (in x-space) of the voltage distribution and the basic charge distribution function (BCDF); the BCDF is now known exactly from (7).

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