Nodal Analysis of Multitransducer SAW Devices

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Abstract — Unified approach to the nodal analysis of multitransducer surface acoustic wave (SAW) devices is presented. Assumed for a grounded SAW transducer to be reflectionless, analytic formulae for the nodal admittance matrix (NAM) interrelating currents and voltages on the electric ports are deduced. Diagonal NAM elements represent self-admittances of SAW transducers and can be calculated by applying the known techniques of the SAW transducer admittance calculation. Off-diagonal elements accounting for acoustic interaction of the elemental SAW transducers are deduced from the short-circuit currents induced in each grounded SAW transducer when only one SAW transducer of the system is activated, with all the others being grounded. The closed-form NAM allows to extend standard nodal analysis techniques for the electric networks to the acousto-electric circuits comprising both SAW devices as well as conventional electric components.

I. INTRODUCTION

SAW technology has greatly evolved over the last decade and many new SAW device applications have emerged such as mobile and satellite communications, ID-tags, etc. While there are applications where conventional SAW bandpass filters comprising two bidirectional interdigital transducers (IDT) can be effectively used it is a modern trend to incorporate into systems innovated SAW devices, such as low-loss SAW filters with interdigitated IDT, image-connected impedance SAW filters, ladder-type filters, etc. having more than two SAW transducers. Design and simulation tools for multitransducer SAW devices are much more complicated and time consuming if compared to the conventional SAW bandpass filters.

The standard approach to their modelling is as follows. Given the mixed scattering matrix (P-matrix) [1, 2] for each separate SAW component, the overall system matrix can be calculated by cascading P-matrices or by applying signal-flow-graph analysis [3]. However, complicated calculations including matrix inversions and/or solving linear equation system are involved to determine the overall system matrix at each frequency.

Alternative analysis approach is considered in the present paper with a basic assumption that a short-circuit (grounded) SAW transducer is reflectionless. The nodal analysis technique is applied instead of cascading, with the nodal admittance matrix deduced analytically using the known mixed scattering matrices of the elemental SAW transducers. Once the nodal admittance matrix is determined, standard nodal analysis techniques and software developed for the electric network analysis can be directly applied to the complicated acousto-electric circuits comprising both multitransducer SAW devices as well as conventional electric components.

II. NODAL ANALYSIS OF A SAW MULTITRANSDUCER SYSTEM

Statement of the problem

Consider an arbitrary SAW multitransducer system with multiple input/output electric ports in general case (Fig. 1).

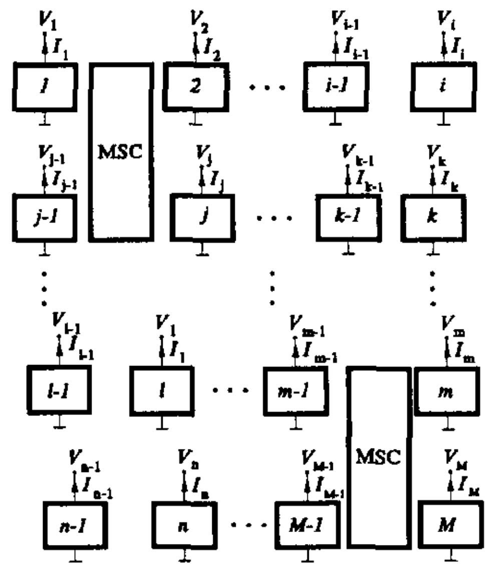

Fig. 1. Hypothetic SAW multitransducer system

SAW transducers are supposed to be uniform (unapodized) or aperture-weighted (apodized) and they may be combined in an arbitrary multitrack configuration using

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multistrip couplers if necessary to couple acoustically two different tracks. The number of tracks, the number of SAW transducers in each track, and positions of SAW transducers within a track are arbitrary. SAW transducers are coupled acoustically either directly when cascaded in-line or via multistrip couplers (with full or partial track coupling) connecting the adjacent acoustic tracks.

The problem is to determine the currents I_i and voltages V_k at the electric ports taking into account acoustic interactions between acoustic ports.

The unified approach to solving the problem based on the nodal analysis of the SAW multitransducer system is considered in this paper where the concept of the nodal admittance matrix (NAM) is applied to an arbitrary SAW multitransducer system, with the closed-form NAM elements deduced. Once the NAM of a SAW system is known, the analysis problem is reduced to the standard nodal analysis of the electric multiport network.

Three-port representation and mixed scattering matrix of a single SAW transducer

Each elemental SAW transducer of a system may be considered as a reciprocal and lossless three-port device with one electric port and two acoustic ports (Fig. 2) where V_k is the voltage applied to the transducer bus-bars, I_k is the transducer terminal current, and $a_k^{(i)}$, $b_k^{(i)}$, $i=1,2$ denote incident (incoming) and reflected (outcoming) waves at the acoustic ports.

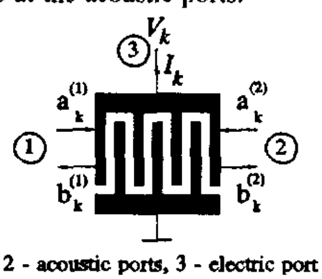


Fig. 2. Three-port representation of the k-th SAW transducer

It is convenient to characterize a SAW transducer as a three-port device by its mixed scattering matrix (P-matrix) [1, 2] interrelating reflected wave amplitudes $b_k^{(i)}$ and the terminal current I_k with the incident wave amplitudes $a_k^{(i)}$ and the applied voltage V_k as follows

$$\begin{bmatrix} b_k^{(1)} \\ b_k^{(2)} \\ I_k \end{bmatrix} = \begin{bmatrix} r_k & t_k & m_k \\ t_k & r_k & m_k^* \\ m_k & m_k^* & Y_k \end{bmatrix} \begin{bmatrix} a_k^{(1)} \\ a_k^{(2)} \\ V_k \end{bmatrix} \qquad (1)$$

where r_k and t_k are reflection and transduction factors of the short-circuit transducer ($V_k=0$), m_k is the electroacoustic transfer function, and Y_k is the transducer admittance. All the mixed scattering matrix elements in Eq. 1 are referred to the transducer center and it is assumed that the matrix is symmetric due to the appropriate acoustic wave amplitude normalization.

When the central frequency of a SAW transducer is far away of the synchronous frequency it may be presumed with a good accuracy that the short-circuit SAW transducer is reflectionless ($r_k=0$, $t_k=1$) and this is a basic assumption of our approach. This assumption simplifies greatly a nodal analysis and closed-form expressions for the NAM elements can be deduced in this case.

Interrelation of currents and voltages in a SAW multitransducer system

When separate SAW transducers are combined in an arbitrary multitransducer system (Fig. 1) currents I_i and voltages V_k at the electric ports are interrelated via a NAM with the elements Y_{ik} as follows

$$\mathbf{I}=\mathbf{Y}\,\mathbf{V} \qquad (2)$$

where **I**, **V** are the vectors of currents and voltages at the electric ports, and **Y** is the square NAM of size M, with M being the total number of the elemental SAW transducers in the system.

Due to the reciprocity property the NAM is symmetric ($Y_{ik}=Y_{ki}$) and due to the causality principle for currents and voltages the real and imaginary part of each element are interrelated via a Hilbert transformation.

Given a mixed scattering matrix for each elemental SAW transducer, the problem is to determine the NAM elements

$$Y_{ik}(\omega)=I_i(\omega)/V_k, \quad V_i=0, \quad i\neq k \qquad (3)$$

where $I_i(\omega)$ is the short-circuit current induced in the i-th transducer when only the k-th transducer of the system is activated by applying the voltage V_k to the transducer basbars, with all the others being short-circuit (grounded).

Assumed for the short-circuit SAW transducer to be reflectionless ("transparent") with respect to the incident waves, there are no reflected waves in the SAW system where only the k-th SAW transducer is activated (Fig. 3). Therefore, the excited waves $b_k^{(i)}$ remain the only waves propagating in each direction to induce short-circuit currents $I_i(\omega)$ when passing past each grounded transducer.

Calculation of self-admittances

As there are no reflected waves in the short-circuit

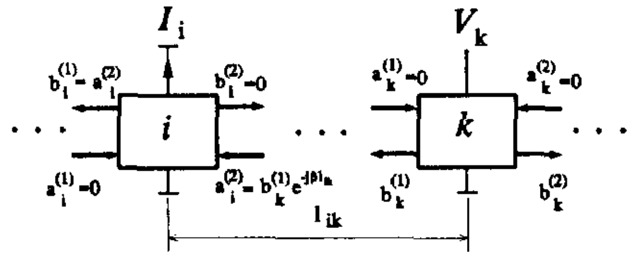


Fig. 3. Acoustic interaction of the i-th and k-th elemental SAW transducers

system the diagonal NAM elements or self-admittances coincide with the admittances $Y_i(\omega)$ of the elemental SAW transducers

$$Y_{ii}(\omega)=Y_i(\omega)=I_i(\omega)/V_i \tag{4}$$

It has been shown [4,5] that in the quasi-static approximation [2] the SAW transducer admittance is the weighted sum of the interelectrode admittances

$$Y(\omega)=\sum_{p=1}^{N-1} L_p y_p(\omega)\ ,\quad L_p=\sum_{k=0}^{N-p-1} W_{k,k+p} \tag{5}$$

where $y_p(\omega)$ are interelectrode admittances comprising both radiation conductance and susceptance as well as the electrostatic component, $W_{k,k+p}$ are overlaps of the k-th and $(k+p)$-th electrodes, and quantities L_p are effective apertures defined by the total overlaps of all the nearest, next nearest neighbour electrodes, and so on, respectively. Closed-form formulae for the interelectrode admittances $y_p(\omega)$ can be found elsewhere [4,5].

Calculation of mutual intertransducer admittances

As there are no reflections within short-circuit system there are no incident waves at the acoustic ports of the k-th SAW transducer ($a_k^{(1)}=a_k^{(2)}=0$). Only the waves $b_k^{(1)}$ and $b_k^{(2)}$ are excited and propagate in both directions. Therefore, for each direction the only wave incident to one of the acoustic ports of each grounded SAW transducer is

$$a_i^{(2)}=b_k^{(1)}e^{-j\beta l_{ik}},\quad i<k \tag{6}$$

or

$$a_i^{(1)}=b_k^{(2)}e^{-j\beta l_{ik}},\quad i>k \tag{7}$$

where l_{ik} is separation between the centers of the i-th and k-th transducers, $\beta=\omega/\mathrm{v}$ is the SAW wavenumber. Using the mixed scattering matrix (1) to express the wave amplitudes $b_k^{(1)}$ and $b_k^{(2)}$ as well as the short-circuit current $I_i(\omega)$ induced in the i-th transducer by the incident waves (6) or (7) we obtain the following closed-form expression for the mutual intertransducer admittances

$$Y_{ik}(\omega)=Y_{ki}(\omega)=\begin{cases} m_i^* m_k e^{-j\beta l_{ik}}, & i<k \\ m_i m_k^* e^{-j\beta l_{ik}}, & i>k \end{cases} \tag{8}$$

In quasi-static approximation [2]the electroacoustic transfer function of a periodic SAW transducer is given by the following expression (transducer index is omitted for simplicity)

$$m(\varphi)=\sqrt{\omega W\Gamma}\rho(\varphi)\sum_{n=0}^{N-1} A_n e^{-j(n-\frac{N-1}{2})\varphi},\quad \varphi=\beta d \tag{9}$$

where d is the electrode pitch (period), W is the transducer aperture, $\Gamma=K^2/2\epsilon$ is the piezoelectric constant, with K^2 being the electromechanical coupling factor and ϵ being the effective permittivity of the surface, $\rho(\varphi)$ is the element factor, A_n are the transducer tap-weights, and N is the number of the electrodes.

According to Eq. 8 the intertransducer admittance $Y_{ik}(\omega)$ is defined by the product of the electroacoustic functions of the interacting transducer pair. This expression is valid when at least one of the SAW transducers in the pair is unapodized or two apodized SAW transducers are connected via an ideal multistrip coupler with the unity track-to-track transfer ratio. More accurate multistrip coupler transfer function can be accounted for if necessary [2].

For acoustically uncoupled SAW transducers the appropriate matrix elements are equal to zero $Y_{ik}(\omega)=0$.

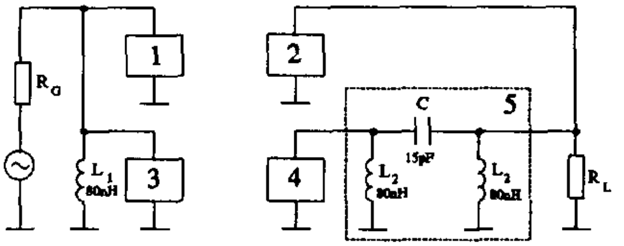
Thus, given for each elemental SAW transducer its admittance $Y_i(\omega)$ and the electroacoustic transfer function $m_i(\omega)$ all the NAM elements $Y_{ik}(\omega)$ are expressed analytically as follows

$$Y_{ik}(\omega)=\begin{cases} Y_i(\omega), & i=k \\ m_i^*(\omega)m_k(\omega)e^{-j\beta l_{ik}}, & i\neq k\ \ (coupled) \\ 0, & i\neq k\ \ (uncoupled) \end{cases} \tag{10}$$

III. SIMULATION EXAMPLE

The nodal analysis technique has been applied to the analysis of the low-loss dual-track SAW filter with the suppressed triple transit echo (TTE) signal (Fig. 4). The SAW filter consists of four SAW transducers. Two output unapodized SAW transducers connected in parallel are identical and they have centrosymmetric finger topology. One of the output apodized SAW transducers is symmetric and the other is antisymmetric with the nearly identical frequency response. Output transducers are spaced at equal distance from the input transducers. However, due to the difference in the symmetry type their transfer functions differ by 90° at any frequency [6].

Therefore, echoes from the output SAW transducers differ by 180° and they are effectively suppressed when incident on the input transducers. Output signals are added constructively due to the additional 90° phase shift (Fig. 4).



1,3 - unapodized IDT (symmetric);
2,4 - apodized IDT (symmetric and antisymmetric);
5 - 90° phase-shifter

Fig. 4. Schematics of the dual-track SAW filter with suppressed triple transit echo signal

The simulation results are shown in Fig. 5 (curve 1) where the ideal filter response (curve 2) calculated using the impulse function model is also shown for comparison. The SAW filter has the central frequency $f_0 = 100$ MHz, 6.7% fractional passband width, and the shape factor of 2 at the -3/-40 dB levels. The substrate material is Y128°X lithium niobate. The electrode numbers in input and output SAW transducers are $N_1 = 53$ and $N_2 = 105$ respectively. Acoustic apertures are $W_1 = W_2 = 1.8$ mm, metallization ratio is 0.5. The synchronous frequency is $f_s = 2f_0$ that corresponds transducer structures with splitted fingers.

The parameters of the electric components used for matching and phase shifting are the following: the capacitor $C = 14$ pF, the inductors $L_1 = L_2 = 0.08$ μH. The source and load impedances are $R_G = R_L = 50$ Ω.

The predicted insertion loss value is -7 dB and the analysis shows nearly total TTE suppression in the filter passband.

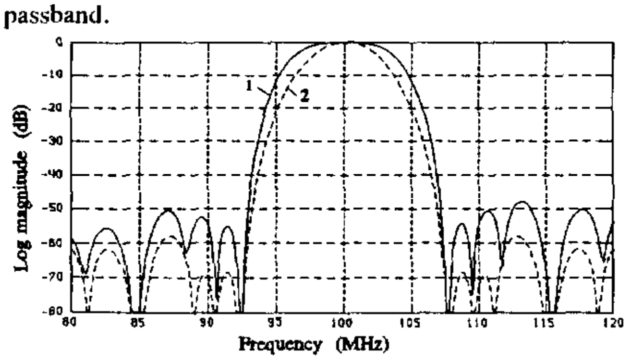


Fig. 5. Frequency response of the dual-track filter: 1 - modelled, 2 - ideal

IV. CONCLUSION

Supposed for a grounded SAW transducer to be reflectionless, the closed-form nodal admittance matrix of an arbitrary SAW multitransducer system has been deduced. The diagonal matrix elements (self-admittances) are defined by the admittances of the elemental SAW transducers of the system. The off-diagonal elements (mutual intertransducer admittances) are defined as the product of the electroacoustic transfer functions of the interacting SAW transducer pairs. Multistrip coupler transfer ratio with full or partial track coupling can be also accounted for if necessary.

The proposed approach allows to extend standard nodal analysis of the electric networks to the acousto-electric systems comprising both multitransducer SAW devices as well as the conventional electric components used for matching for example.

Within the basic SAW model constraint applied (non-reflectivity of a short-circuit SAW transducer) multiple interactions of acoustically coupled SAW transducers as well their interactions with the external electric components are fully taken into account in such a nodal analysis. Once the nodal admittance matrix has been determined for the system, the standard nodal analysis techniques and software developed for electric network analysis can be used.

The proposed approach can be effectively applied to the modelling of non-resonant SAW devices having the central frequency away from the synchronous frequency.

REFERENCES

[1] G.Tobolka, "Mixed matrix representation of SAW transducers," *IEEE Trans. Sonics. Ultrason.*, vol. SU-26, No. 6, pp. 426-428, 1979.

[2] D.P.Morgan, *Surface-wave devices for signal processing*, Elsevier, Amsterdam, 1985.

[3] O.Manner, K.C.Wagner, and W.Ruile, "Signal flow graph analysis of diffraction and reflection in SAW filters," in *IEEE Ultrason. Symp. Proc.*, 1992, pp. 1-5.

[4] A.S.Rukhlenko, "Charge distribution and capacitance calculation for generalized periodic SAW transducers using Floquet's technique," in *IEEE Ultrason. Symp. Proc.*, 1994, pp. 325-329.

[5] A.S.Rukhlenko, "Closed-form admittance calculation for generalized periodic SAW transducers," to be published in *1995 World Congress Ultrason. Proc.*, 1995, Berlin.

[6] R.I.Murray, R.C.Peach, and S.Neylon, "The quadrature taps for SAW delay lines," Electron. Lett., 1978, vol. 14, pp. 631-632.