# Closed-Form Modeling of Reflective SAW Transducers with Arbitrary Polarity Sequence and Apodization

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*Abstract* – **A new closed-form phenomenological model of reflective polarity-weighted or apodized surface acoustic wave (SAW) transducers has been developed. The results are presented in the form of the mixed scattering matrix comprising the acoustoelectric conversion function and transducer admittance. The analogy between acoustic wave and transmission line equations has been used to deduce the closed-form equations with two principal model parameters: 1) the wave propagation constant to be found from the dispersion equation and 2) the coupling factor between the fundamental and first backward spatial harmonics. The classical COM equations follow from the model as the particular case of the uniform solid-finger SAW transducer with a regular sign alternation. In the case of reflectionless SAW transducers, equations are reduced to the quasi-static approximation. Modeling results for solid-finger withdrawal-weighted and apodized SAW transducers are presented which agree well with the published results and experimental data.**

## I. INTRODUCTION

Modeling of reflective surface acoustic wave (SAW) transducers when the central frequency is close to the synchronous (Bragg) frequency is important in many SAW devices. For uniform periodic transducers, the closed-form equations have been derived using the coupling-of-modes (COM) formalism [1-2]. Due to simplicity and accuracy, the COM model is widely used in the computer-aided design of SAW devices. However, it cannot be directly applied to the modeling of unapodized SAW transducers with an arbitrary polarity sequence or apodization.

An alternative approach to model reflective SAW transducers is the reflective array modeling (RAM) [3, 4] which is based on cascading the elemental cells in terms of the mixed scattering matrices (P-matrices) [5]. This numeric technique can be applied to SAW transducers with arbitrary polarity sequences. However, for long transducers with many fingers, it involves extensive computations comprising recurrent matrix operations.

Some other techniques have been developed to account for interelectrode reflections [6-10]. However, they did not come to practice due to insufficient accuracy or theoretical complexity.

Therefore, the closed-form model combining advantages of the COM model and quasi-static approximation is desirable.

This paper is concerned with the closed-form modeling of generalized (withdrawal-weighted or apodized) SAW transducers. A principal assumption is that finger reflectivity is uniform throughout the structure, i.e. the finger width $a$ and period $p$ are constant (Fig. 1, a) and, hence, the finger reflection coefficient is the same for all electrodes. Based on the analogy between COM and transmission line equations, the phenomenological closed-form model has been developed. A complete mixed scattering matrix [5] has been deduced including the acoustoelectric function and transducer admittance. The model is valid for all frequencies of interest and for different types of transducer periodicity (solid-finger, split-finger, etc.). The model accounts for interelectrode reflections due to mass-electrical loading and reflections at the transducer edges (the boundary between the transducer region and the free surface of the substrate).

## II. MODELING APPROACH

### A. Statement of the Problem

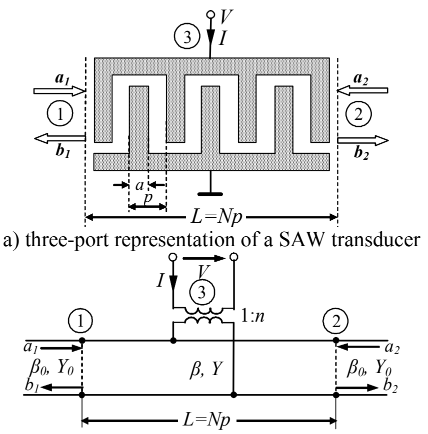
We consider a periodic SAW transducer with the finger period (pitch) $p$, the number of fingers $N$, and the metallization ratio (duty factor) $\eta = a/p$ where $a$ is the finger width (Fig. 1, a). In general case, the transducer can be withdrawal (polarity)-weighted or apodized. In the transducer region, the SAW wavenumber is $\beta < \beta_0$ where $\beta_0 = \omega/v_0$ is the free surface wavenumber, $\omega = 2\pi f$ is the cycle frequency, $v_0$ is the free surface SAW velocity. The transducer has two acoustic ports 1 and 2 and one electric port 3 which is used for electrical excitation. The problem is to determine the mixed scattering matrix [5] of the transducer taking into account the finger reflectivity.

### B. Passive Grating

We shall start our consideration from the case where there is no voltage applied to the transducer bus-bars, i. e. all fingers are grounded ($V$=0). A short-circuit SAW transducer is virtually the passive periodic grating that scatters incident acoustic waves. The principal assumption is that the major contribution to the acoustic power carried in the forward and backward directions by the counter-propagating acoustic waves in the periodic grating is given by the fundamental and first negative spatial harmonics in the Floquet's modal expansion [11]

$$\begin{aligned} a^+(x) &= a_0^+ e^{-j\beta x} + a_{-1}^+ e^{j(K-\beta)x} = a_0^+ (e^{-j\beta x} + \gamma e^{j(K-\beta)x}) \\ a^-(x) &= a_0^- e^{j\beta x} + a_{-1}^- e^{-j(K-\beta)x} = a_0^- (e^{j\beta x} + \gamma e^{-j(K-\beta)x}) \end{aligned} \quad (1)$$

where $\beta$ is the fundamental wavenumber, $K$=2π/p is the grating wavenumber, $\gamma = a_{-1}^{\pm} / a_0^{\pm}$ is the coupling factor between the fundamental and first negative spatial harmonics, $a_0^{\pm}$ and $a_{-1}^{\pm}$ are the fundamental and first harmonic amplitudes for the forward (+) and backward (–) propagating waves, respectively.



a) three-port representation of a SAW transducer

b) equivalent transmission line circuit

Fig. 1. Equivalent circuit of a SAW transducer

By matching the incident waves $a_1$=1 and $a_2$=0 at the grating ends with the modal waves in (1) having phase velocity in the same direction we obtain the scattering coefficients [11]

$$S_{11} = S_{22} = \gamma \frac{1 - e^{j(K-2\beta L)}}{1-\gamma^2 e^{j(K-2\beta L)}}, \quad S_{12} = S_{21} = \frac{(1-\gamma^2)\, e^{-j\beta L}}{1-\gamma^2 e^{j(K-2\beta L)}} \qquad (2)$$

where $L=Np$ is the grating length, $N$ is the number of the strips, and β is to be found from the dispersion equation. We slightly modify (2) by offsetting outward the acoustic port reference plane by $p/2$ from the strip center (that changes the sign of the reflection coefficient $S_{11}$) and by omitting multiples of $Kp=2\pi$

$$S_{11} = -\gamma \frac{1-e^{-2N}}{1-\gamma^2 e^{-2N}}, \quad S_{12} = \frac{\tau^2 e^{-N}}{1-\gamma^2 e^{-2N}}, \quad \tau^2 = 1-\gamma^2 \qquad (3)$$

where $e=e^{j\varphi}$, $\varphi=\beta p$, for brevity. These equations are well-known in the COM theory [1-2]. However, no analogy between these equations and transmission line equations has been discovered.

We consider an equivalent circuit constructed by the three transmission line (TL) segments (Fig. 1, b). The electric port 3 is short-circuit and omitted. The central TL of the length $L$ has the characteristic admittance $Y$ and the propagation constant β. The identical semi-infinite TL with the characteristic admittance $Y_0$ and the wavenumber $\beta_0$ represent the free surface aside. The central TL segment (transducer region) and jump discontinuities (mismatched TL) at the boundaries between the free surface and transducer are described by their scattering matrices [12]

$$\mathbf{S}_0 = \begin{bmatrix} 0 & e^{-N} \\ e^{-N} & 0 \end{bmatrix}, \quad \mathbf{S}^{\pm} = \begin{bmatrix} \pm\gamma & \theta \\ \theta & \mp\gamma \end{bmatrix}, \qquad (4)$$

where $\gamma = (1-n^2)/(1+n^2)$, $\theta = 2n/(1+n^2)$, $\gamma^2+\theta^2=1$, $n^2 = Y/Y_0$. The lower and upper signs are referred to the left and right acoustic ports 1 and 2, respectively. Here, γ and θ stand for the reflection and transmission coefficients and $n^2$ is the admittance transformation, or mismatch, ratio.

For cascading it is convenient to convert the scattering matrices $\mathbf{S}_0$ and $\mathbf{S}^{\pm}$ which relates the reflected waves $b_i$, $i$=1,2 with the incident waves $a_i$ to the transmission matrices $\mathbf{T}_0$ and $\mathbf{T}^{\pm}$ relating the waves $a_1$, $b_1$ at the acoustic port 1 (input) and the waves $a_2$, $b_2$ at the acoustic port 2 (output). We apply the following interrelation between the scattering and transmission matrices

$$\mathbf{T} = \begin{bmatrix} \dfrac{1}{S_{21}} & -\dfrac{S_{22}}{S_{21}} \\ \dfrac{S_{11}}{S_{21}} & S_{12} - \dfrac{S_{11}S_{22}}{S_{21}} \end{bmatrix}, \quad \mathbf{S} = \begin{bmatrix} \dfrac{T_{21}}{T_{11}} & T_{22} - \dfrac{T_{12}T_{21}}{T_{11}} \\ \dfrac{1}{T_{11}} & -\dfrac{T_{12}}{T_{11}} \end{bmatrix}. \qquad (5)$$

The overall transmission matrix can be found as $\mathbf{T}=\mathbf{T}^{-}\mathbf{T}_0\mathbf{T}^{+}$. After cascading, we can recover the overall scattering matrix $\mathbf{S}=[S_{ik}]$, $i,k$=1,2 using (5) which has exactly the same elements (3). Therefore, the analogy between wave scattering in the grating and in the equivalent circuit has been proven.

### *C. Harmonic Coupling Factor*

The harmonic coupling factor γ is the ratio of the amplitudes of the first negative and fundamental spatial harmonics [11]. In COM theory [1-2] the value of γ is approximated as $\gamma=\kappa/(\delta+k)$ where $\delta=\beta_0+\Delta\beta-K/2$ is the detuning parameter, $\beta_0$ is the free surface wavenumber, Δβ is the wavenumber perturbation, $k = \beta - K/2 = \sqrt{\delta^2 - |\kappa|^2}$ is the propagation constant, and κ is the finger reflectivity per unit length.

The more rigorous equation for γ can be deduced by taking limit of (3) at $N\to\infty$ that results in $\gamma=-S_{11}^{\infty}$ where $S_{11}^{\infty}$ is the reflection coefficient of the semi-infinite grating. In the limit case of Morgan's RAM [4] we obtain

$$\gamma = -\lim_{N\to\infty} \frac{r}{t} \frac{\sin N\varphi}{\sin N\varphi - \tau \sin(N-1)\varphi} = -\frac{\rho}{1-\tau e^{-1}} \qquad (6)$$

where $\rho = re^{j\varphi_0}$, $\tau = te^{j\varphi_0}$, $\varphi_0 = \beta_0 p$, $\varphi = \beta p$ and $r$, $t$ are the finger reflection and transmission coefficients referred to the finger center. The identity $\lim_{N\to\infty} e^{-jN\varphi} = 0$ has been used to deduce (6). Therefore, the harmonic coupling coefficient γ is related with the scattering coefficients $r$ and $t$ of the elemental cell.

### *D. Propagation Constant*

The propagation constant β needs to be found from the dispersion equation for the periodic infinite grating. In the COM theory [1-2], its value is approximated as

$$\beta = K/2 + \sqrt{\delta^2 - |\kappa|^2} \qquad (7)$$

The propagation constant β is the real wavenumber in the grating passband ($|\delta| \geq |\kappa|$) and it has the imaginary part in the grating stopband ($|\delta| < |\kappa|$) resulting in SAW attenuation.

More rigorous solution has been deduced in [4] where β is found from the dispersion equation

$$\cos\beta p = \frac{1}{2}\left(\frac{1}{\tau} + \frac{1}{\tau^*}\right) = \frac{\cos(\beta_0 p)}{|t|} \qquad (8)$$

It can be shown [4] that the approximation (7) follows from (8) in the particular case when the frequency is close to the synchronous frequency $f_\pi=v/2p$.

## III. CLOSED-FORM REFLECTIVE TRANSDUCER MODEL

The analogy between wave and TL equations leads to the following interpretation of the wave propagation in a SAW transducer. According to (4), there are two counter-propagating “free” (uncoupled) waves in the transducer region (central section). The propagation constant β accounts for the wave interaction with the grating from the solution of the dispersion equation (8). The two mismatched TL at the transducer ends account for reflections due to acoustic mismatch between the transducer region and free surface of the substrate.

We generalize by a phenomenological way the passive grating model to a SAW transducer with the voltage $V$ applied to the bus-bars. By analogy with the “free” wave propagation (4), we describe the transducer region using the “quasi-static” approximation with two principal modifications:

1) the complex-valued propagation constant β is taken instead of the conventional wavenumber $\beta=\omega/v$,

2) the characteristic admittance is transformed in accordance with the admittance mismatch ratio $n^2=Y/Y_0$.

The equivalent circuit sections are characterized by the following matrices

$$\mathbf{M}_0=\begin{bmatrix}0 & e^{-N} & nm_{13}\\ e^{-N} & 0 & nm_{23}\\ nm_{31} & nm_{32} & n^2m_{33}\end{bmatrix},\mathbf{M}^{\pm}=\begin{bmatrix}\pm\gamma & \theta & 0\\ \theta & \mp\gamma & 0\\ 0 & 0 & 1\end{bmatrix}. \qquad (9)$$

where $n^2=Y/Y_0=(1+\gamma)/(1-\gamma)$ is the admittance mismatch ratio, $\mathbf{M}_0$ is the “quasi-static” mixed scattering matrix. The two-port scattering matrices $\mathbf{S}^{\pm}$ have been replaced by the augmented three-port mixed scattering matrices $\mathbf{M}^{\pm}$. Here, $m_{i3}$, $i$=1, 2 are the acoustoelectric functions, $m_{3i}=-2m_{i3}$, $i$=1, 2 are the electroacoustic functions, and $m_{33}$ is the transducer admittance.

Since the central acoustoelectric section (Fig. 1, b) is loaded at both ends by the mismatched TL with the mixed scattering matrices $\mathbf{M}^{\pm}$, the overall mixed scattering matrix $\mathbf{M}$ can be found by cascading the constitutive TL segments using the following relations between the mixed scattering and transmission matrices

$$\mathbf{T}=\begin{bmatrix}\dfrac{1}{M_{21}} & -\dfrac{M_{22}}{M_{21}} & -\dfrac{M_{23}}{M_{21}}\\ \dfrac{M_{11}}{M_{21}} & M_{12}-\dfrac{M_{11}M_{22}}{M_{21}} & M_{13}-\dfrac{M_{11}M_{23}}{M_{21}}\\ \dfrac{M_{31}}{M_{21}} & M_{32}-\dfrac{M_{22}M_{31}}{M_{21}} & M_{33}-\dfrac{M_{31}M_{23}}{M_{21}}\end{bmatrix},\ \mathbf{M}=\begin{bmatrix}\dfrac{T_{21}}{T_{11}} & T_{22}-\dfrac{T_{12}T_{21}}{T_{11}} & T_{23}-\dfrac{T_{21}T_{13}}{T_{11}}\\ \dfrac{1}{T_{11}} & -\dfrac{T_{12}}{T_{11}} & -\dfrac{T_{13}}{T_{11}}\\ \dfrac{T_{31}}{T_{11}} & T_{32}-\dfrac{T_{12}T_{31}}{T_{11}} & T_{33}-\dfrac{T_{13}T_{31}}{T_{11}}\end{bmatrix} \qquad (10)$$

The three-port mixed matrixes $\mathbf{M}_0$ and $\mathbf{M}^{\pm}$ need to be converted to the three-port transmission matrices $\mathbf{T}_0$ and $\mathbf{T}^{\pm}$ using (10). The overall transmission matrix $\mathbf{T}=\mathbf{T}^{-}\mathbf{T}_0\mathbf{T}^{+}$ is given by

$$\mathbf{T}=\frac{1}{\theta^2}\begin{bmatrix}(1-\gamma^2e^{-2N})e^{N} & \gamma(1-e^{-2N})e^{N} & -n\theta(m_{23}e^{N}+\gamma m_{13})\\ -\gamma(1-e^{-2N})e^{N} & (1-\gamma^2e^{2N})e^{-N} & n\theta(m_{13}+\gamma m_{23}e^{N})\\ n\theta(m_{31}e^{N}+\gamma m_{32}) & n\theta(m_{32}+\gamma m_{31}e^{N}) & n^2\theta^2(m_{33}-m_{31}m_{23}e^{N})\end{bmatrix} \qquad (11)$$

where $\theta^2=1-\gamma^2$. The elements of the mixed scattering matrix $\mathbf{M}=[M_{ik}]$, $i,k$=1,2,3 are reconstructed from (11) by using (10):

$$\begin{aligned}&M_{11}=M_{22}=-\gamma\frac{1-e^{-2N}}{\Delta},\ M_{12}=M_{21}=\frac{1-\gamma^2}{\Delta}e^{-N},\ \Delta=1-\gamma^2e^{-2N},\\ &M_{13}=\frac{1+\gamma}{\Delta}(m_{13}+\gamma m_{23}e^{-N}),\ M_{23}=\frac{1+\gamma}{\Delta}(m_{23}+\gamma m_{13}e^{-N}),\\ &M_{31}=-2M_{13},\ M_{32}=-2M_{23},\ M_{33}=\frac{1+\gamma}{1-\gamma}\left(m_{33}+2m_{13}m_{23}e^{N}\right)-2\frac{M_{13}M_{23}}{M_{21}}\end{aligned} \qquad (12)$$

The equations (12) account for rigorously all multipath reflections due to interelectrode interactions and acoustic admittance discontinuities at the transducer edges. These equations are general and can be used for withdrawal-weighted and apodized SAW transducers. In the particular case of γ=0, they are reduced to the quasi-static approximation [4]. All the elements of the mixed scattering matrix (12) including the transducer admittance can be determined in the closed-form [4, 13, 14].

## IV. MODEL VERIFICATION AND EXAMPLES

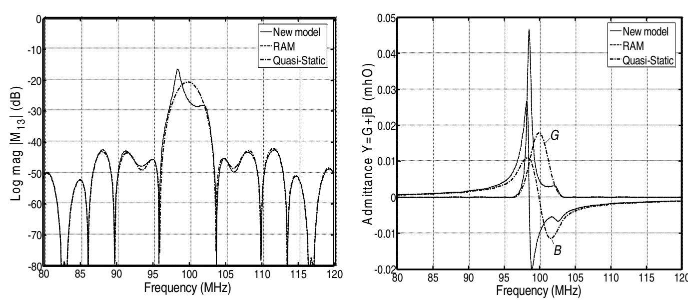
### *A. Solid Finger Uniform Transducer*

The proposed phenomenological model has been verified both analytically and numerically by comparison with the known models, published results, and experimental data.

For uniform solid finger SAW transducers the closed-form mixed scattering matrix elements are available in the quasi-static approximation [4]. By substituting these results into (12) it can be shown that for this particular case equations (12) can be reduced to conventional COM equations [1-2]. Therefore, the proposed model is in the perfect agreement with the COM model providing much more generality and flexibility.

### *B. Withdrawal-Weighted Transducer*

The modeled results (the acoustoelectric function $M_{13}(\omega)$ and the transducer admittance $Y(\omega)=G(\omega)+jB(\omega)$ ) have been compared with Morgan’s RAM for the withdrawal-weighted SAW transducer [3] (Fig. 2). Both models give identical results. The quasi-static characteristics (no reflections, $r$=0, $t$=1) are also shown in Fig. 2, for comparison.




a) acoustoelectric function b) transducer admittance

Fig. 2. Modeled results for the withdrawal-weighted SAW transducer [3]

### *C. Apodized Transducer*

The new model was compared with the published simulation and experimental data [4] where the closed form equation for the acoustoelectric conversion function has been deduced in the particular case of the symmetric apodization. The modeling results are presented in Fig. 3. The number of fingers in the apodized SAW transducer is $N$=51, the substrate material is YZ $LiNbO_3$, the finger reflection coefficient is $r$=-0.015$j$. There is perfect correspondence between our model and RAM and good correspondence of both models with the experimental data.

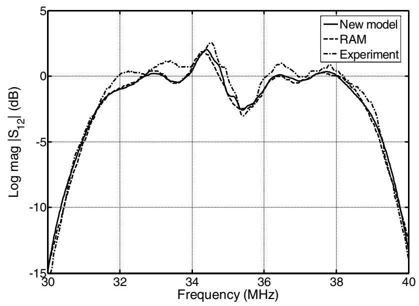
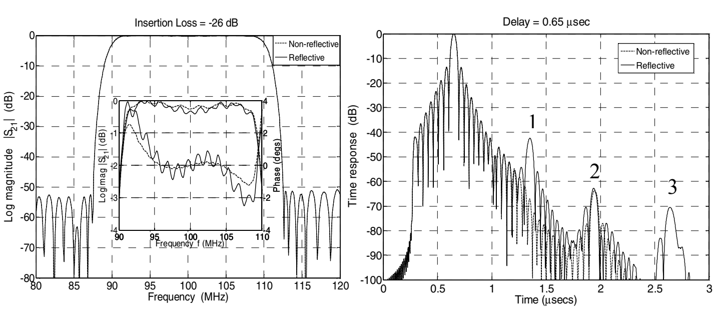



Fig. 3. Model comparison with the published theoretical results and experimental data for the apodized SAW transducer

### *D. Split-Finger SAW Filter Simulation*

It is known that even in the split-finger SAW filter design there are residual SAW intertransducer reflections for the high coupling piezoelectric substrates (e.g. $LiNbO_3$) [15]. However, these effects were not modeled in the wide practice as the existing models require extensive computations for the apodized SAW transducers. In Fig. 4 the modeled results are shown in the quasi-static approximation (no reflections) and using the proposed closed-form model. The substrate material is $128^0$ LiNbO3. The SAW filter has $N_1$=28 and $N_2$=280 fingers in the input unapodized and output apodized SAW transducers, respectively. The SAW filter aperture is $W$=1 mm, the metallization ratio $\eta$=0.5, the gap between transducers is 1 mm. The finger reflection coefficient is taken as $r$=-0.016$j$.

As can be seen in Fig. 4, a, the intertransducer reflections increase significantly the passband magnitude ripple (from less than 0.3 to about 0.5 dB) and phase ripple (from 3 to 6 degrees). In Fig. 4, b the multipath time domain intertransducer reflections 1 and 3 giving the most significant contribution to the frequency response can be recognized while the sidelobe 2 is attributed to the regenerated triple transit echo signal.




a) frequency response b) time response

Fig. 4. Split-finger SAW filter modeling

## V. CONCLUSIONS

A new closed-form phenomenological model of reflective polarity-weighted and/or apodized SAW transducers has been developed. The results are presented in the form of the complete mixed scattering matrix comprising the acoustoelectric conversion function and transducer admittance (both radiation conductance and susceptance). The well-known COM-equations follow from this model as a particular case. In another particular case of the non-reflective transducer, the results are reduced to the conventional quasi-static approximation.

The analysis results for solid finger withdrawal-weighted and apodized SAW transducers are presented which agree well with other models and experimental data.

The model can be generalized to periodic NSPUDT transducers and LSAW substrates that is to be done next.

## ACKNOWLEDGMENT

The author thanks Dr. D. P. Morgan for reading the manuscript and making valuable comments and corrections.

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