MODELING OF MULTIPORT SURFACE ACOUSTIC WAVE DEVICES

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Abstract

General approach to the analysis of multiport/multitransducer surface acoustic wave (SAW) devices is presented, provided for the scattering matrices of SAW components to be known a priori. Active SAW components such as SAW transducers are described in terms of the mixed electroacoustic scattering matrices (P-matrices) while passive SAW components (multistrip couplers, reflective gratings, etc.) are characterized by the acoustic wave scattering matrices Scattering matrices of SAW components can be deduced by applying the known SAW modeling techniques (equivalent scheme method, COM-analysis, etc) The solution of the problem is obtained in the block-matrix form. Separation of the tasks (a priori modeling of SAW components and modeling of the overall SAW system) provides excellent up-grade ability with respect to the modeling complexity and accuracy

Statement of the problem

Consider an arbitrary surface acoustic wave (SAW) system with multiple acoustic and electric ports which may comprise various SAW components (SAW transducers, multistrip couplers, reflecting gratings, etc.), in general case (Fig. 1).

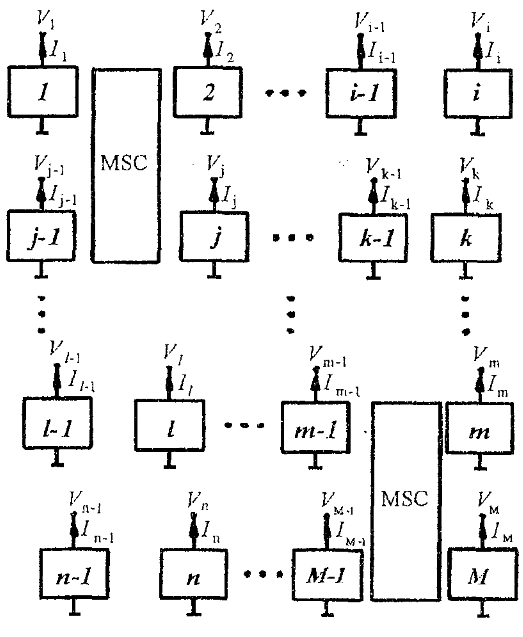

Fig. 1. SAW multiport/multitransducer system

SAW components are combined in an arbitrary multitrack configuration. The number of tracks, the number of SAW components in each track and their positions are arbitrary. SAW components may be coupled acoustically either directly when cascaded in-line or via multistrip couplers (with full or partial track coupling) to connect the adjacent acoustic tracks.

The problem is to determine the currents I_i and voltages V_k at the initial electric ports taking into account all acoustic interactions between acoustic ports in a SAW system

The unified approach to solving the problem based on the nodal analysis of a SAW multitransducer system in the quasi-static approximation [1] is considered in the paper [2] However, as modern low-loss SAW filters for mobile communication may use diversity of acoustic resonant components such as reflective gratings, image-impedance connected SAW transducers, track-changers, etc. [3], the quasi-static approximation is no longer valid for modeling such devices The cascading method [4] is convenient for computer-aided design but it does not give the closed-form solution of the problem

Two different closed-form matrix techniques are considered in the present paper One of them is close to the method [5] and another refers to the important particular case of the multiport SAW system loaded by two-port SAW components

Scattering matrix approach to the analysis of the multiport SAW system

Assumed for each SAW component to be completely defined by its scattering matrix, we develop here a general approach to deduce overall scattering matrix of the multiport SAW system (Fig. 2). For further convenience, it is assumed that each component is described in terms of a wave scattering matrix. Electric and acoustic variables at the k-th port are interrelated as follows

$$\begin{cases} I_k=\sqrt{Y_k}\,(a_k-b_k) \\ V_k=\sqrt{Z_k}\,(a_k+b_k) \end{cases};\quad \begin{cases} a_k=\frac{1}{2}(\sqrt{Y_k}\,V_k+\sqrt{Z_k}\,I_k) \\ b_k=\frac{1}{2}(\sqrt{Y_k}\,V_k-\sqrt{Z_k}\,I_k) \end{cases} \tag{1}$$

where I_k and V_k are the current and voltage, a_k and b_k are amplitudes of the incident and reflected waves, $Z_k=1/Y_k$ is the characteristic impedance. By applying the Eq. (1) the mixed scattering matrix of a SAW transducer can be converted to the wave scattering matrix and vice versa.

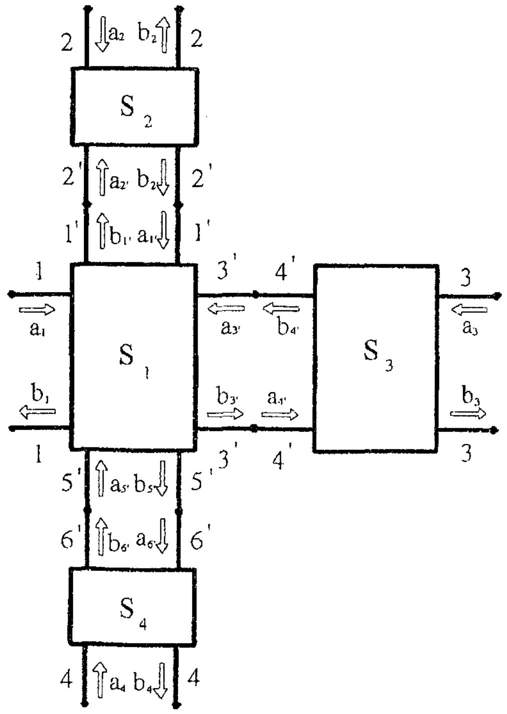

Fig. 2. Wave interaction in an arbitrary multiport SAW system

Compose of the components of the elemental scattering matrices the following composite scattering matrix

$$\bar{\mathbf{S}}=\begin{array}{c|cccc|cccc|}
 & 1 & 2 & 3 & \dots & 1' & 2' & 3' & \dots \\
1 & s_{11} & s_{12} & s_{13} & \dots & s_{11'} & s_{12'} & s_{13'} & \dots \\
2 & s_{21} & s_{22} & s_{23} & \dots & s_{21'} & s_{22'} & s_{23'} & \dots \\
3 & s_{31} & s_{32} & s_{33} & \dots & s_{31'} & s_{32'} & s_{33'} & \dots \\
\vdots & \vdots & & & \vdots & \vdots & & & \vdots \\
\hline
1' & s_{1'1} & s_{1'2} & s_{1'3} & \dots & s_{1'1'} & s_{1'2'} & s_{1'3'} & \dots \\
2' & s_{2'1} & s_{2'2} & s_{2'3} & \dots & s_{2'1'} & s_{2'2'} & s_{2'3'} & \dots \\
3' & s_{3'1} & s_{3'2} & s_{3'3} & \dots & s_{3'1'} & s_{3'2'} & s_{3'3'} & \dots \\
\vdots & \vdots & & & \vdots & \vdots & & & \vdots
\end{array}=\left[\begin{array}{c|c}\bar{\mathbf{S}}_{\alpha\alpha} & \bar{\mathbf{S}}_{\alpha\beta}\\ \hline \bar{\mathbf{S}}_{\beta\alpha} & \bar{\mathbf{S}}_{\beta\beta}\end{array}\right] \tag{2}$$

where the primed indices are attributed to the connected ports. The scattering elements in Eq. (2) are taken from the scattering matrices of SAW components The matrix elements combining indices of two different SAW components are equal to zero.

By the appropriate partition of the scattering matrix (2) we can derive the following block-matrix equation

$$\left[\begin{array}{c}\mathbf{B}_\alpha\\ \hline \mathbf{B}_\beta\end{array}\right]=\left[\begin{array}{c|c}\bar{\mathbf{S}}_{\alpha\alpha} & \bar{\mathbf{S}}_{\alpha\beta}\\ \hline \bar{\mathbf{S}}_{\beta\alpha} & \bar{\mathbf{S}}_{\beta\beta}\end{array}\right]\left[\begin{array}{c}\mathbf{A}_\alpha\\ \hline \mathbf{A}_\beta\end{array}\right] \tag{3}$$

where

$$\mathbf{A}_\alpha=\begin{bmatrix}a_1 & a_2 & a_3 & \dots\end{bmatrix}^{\mathrm{T}};\quad \mathbf{A}_\beta=\begin{bmatrix}a_{1'} & a_{2'} & a_{3'} & \dots\end{bmatrix}^{\mathrm{T}},$$

$$\mathbf{B}_\alpha=\begin{bmatrix}b_1 & b_2 & b_3 & \dots\end{bmatrix}^{\mathrm{T}};\quad \mathbf{B}_\beta=\begin{bmatrix}b_{1'} & b_{2'} & b_{3'} & \dots\end{bmatrix}^{\mathrm{T}}.$$

Now we can impose on the composite matrix (2) the wave coupling conditions. Interrelation between coupled incident and reflected waves at the ports can be expressed via the coupling matrix **F** as follows

$$\mathbf{B}_\beta=\mathbf{F}\ \mathbf{A}_\alpha \tag{4}$$

where the coupling matrix elements are equal to

$$F_{ik}=F_{ki}=\begin{cases}1 & \text{for the coupled } i\text{ - th and } k\text{ - th ports}\\ 0 & \text{otherwise}.\end{cases}$$

For example, the multiport system in Fig. 2 is characterized by the following coupling matrix

$$\mathbf{F}=\begin{array}{c|ccccc|}
 & 1' & 2' & 3' & 4' & \dots \\
1' & 0 & 1 & 0 & 0 & \dots \\
2' & 1 & 0 & 0 & 0 & \dots \\
3' & 0 & 0 & 0 & 1 & \dots \\
4' & 0 & 0 & 1 & 0 & \dots \\
\vdots & \vdots & & & & \vdots
\end{array} \tag{5}$$

By substituting Eq (4) into Eq. (3) and solving the resulting matrix equation system with respect to $\mathbf{B}_\alpha$ we obtain the scattering matrix of the system interrelating the incident and the reflected waves at the independent (uncoupled) acoustic ports

$$\mathbf{B}_\alpha=\mathbf{S}\ \mathbf{A}_\alpha \tag{6}$$

where

$$\mathbf{S}=\mathbf{S}_{\alpha\alpha}+\mathbf{S}_{\alpha\beta}\,(\mathbf{F}-\mathbf{S}_{\beta\beta})^{-1}\ \mathbf{S}_{\alpha\beta}^{\mathrm{T}}\,. \tag{7}$$

The scattering matrix (7) gives the closed-form solution of the problem taking into account all multiple acoustic wave interactions in the multiport SAW system. Once the scattering matrix (7) has been determined, the electric variables at the electric ports can be recovered by applying the Eq (1)

Multiport SAW system loaded by acoustic two-port junctions

The developed matrix approach requires considerable amount of computations in general case To simplify the analysis consider an important particular case when some ports of the multiport SAW system with the scattering matrix $\bar{\mathbf{S}}$ are acoustically or electrically loaded by the two-port devices (SAW reflectors, for example) (Fig 3) which are characterized by the transmission matrices

$$\mathbf{T}_i = \begin{bmatrix} t_{11}^{(i)} & t_{12}^{(i)} \\ t_{21}^{(i)} & t_{22}^{(i)} \end{bmatrix} \tag{8}$$

Each transmission matrix relates incoming and outcoming waves at the i-th port as follows

$$\begin{bmatrix} A_i \\ B_i \end{bmatrix} = \begin{bmatrix} t_{11}^{(i)} & t_{12}^{(i)} \\ t_{21}^{(i)} & t_{22}^{(i)} \end{bmatrix} \begin{bmatrix} a_i \\ b_i \end{bmatrix} \tag{9}$$

where outer (incoming) waves are capitalized

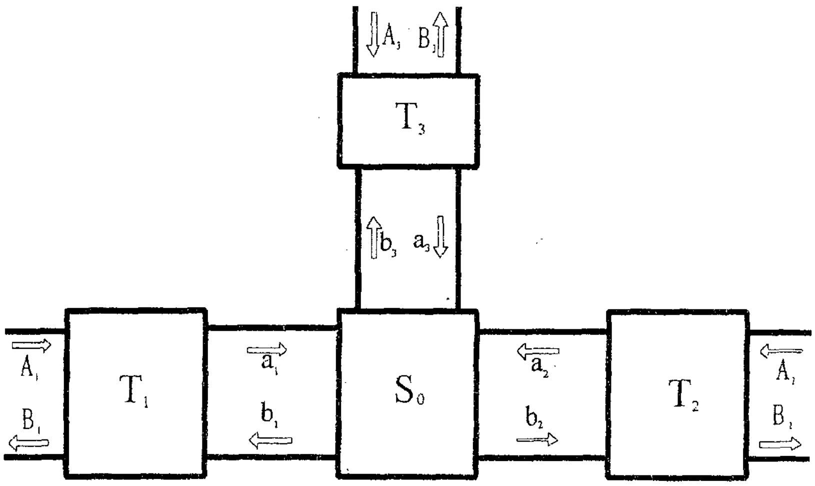

Fig. 3. Multiport SAW system loaded by two-port junctions

For the acoustic ports disconnected from two-port devices, an ideal transmission matrix

$$\mathbf{T}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

should be formally attributed.

All the internal (outcoming) waves are interrelated via the scattering matrix of the form (2)

$$\mathbf{b} = \bar{\mathbf{S}}\,\mathbf{a} \tag{10}$$

where $\mathbf{a}=[a_1\ a_2\ \dots\ a_n]^T$, $\mathbf{b}=[b_1\ b_2\ \dots\ b_n]^T$.

By the appropriate reordering of the vector and matrix elements we can derive the following block-matrix expression interrelating all incoming and outcoming waves of the system

$$\begin{bmatrix} \mathbf{A} \\ \mathbf{B} \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{11} & \mathbf{T}_{12} \\ \mathbf{T}_{21} & \mathbf{T}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} \tag{11}$$

where $\mathbf{A}=[A_1\ A_2\ \dots\ A_n]^T$, $\mathbf{B}=[B_1\ B_2\ \dots\ B_n]^T$ and the block-matrices

$$\mathbf{T}_{ik}=\begin{bmatrix} t_{ik}^{(1)} & 0 & \dots & 0 \\ 0 & t_{ik}^{(2)} & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & t_{ik}^{(n)} \end{bmatrix}, \quad i,k=1,2. \tag{12}$$

By substituting Eq. (10) into Eq. (11) we can express outcoming waves **A** and **B** in terms of the incoming waves **a** as follows

$$\begin{cases} \mathbf{A}=(\mathbf{T}_{11}+\mathbf{T}_{12}\overline{\mathbf{S}})\,\mathbf{a} \\ \mathbf{B}=(\mathbf{T}_{21}+\mathbf{T}_{22}\overline{\mathbf{S}})\,\mathbf{a} \end{cases} \tag{13}$$

After excluding the vector **a** from the Eq. (13), we find the resulting scattering matrix of the system

$$\mathbf{B}=\mathbf{S}\,\mathbf{A}, \quad \mathbf{S}=(\mathbf{T}_{21}+\mathbf{T}_{22}\overline{\mathbf{S}})(\mathbf{T}_{11}+\mathbf{T}_{12}\overline{\mathbf{S}})^{-1} \tag{14}$$

Modeling of the one-port SAW resonator

Apply the developed technique to the modeling of the one-port SAW resonator consisting of the central input/output SAW transducer and two outer SAW reflectors at the both sides (Fig. 4).

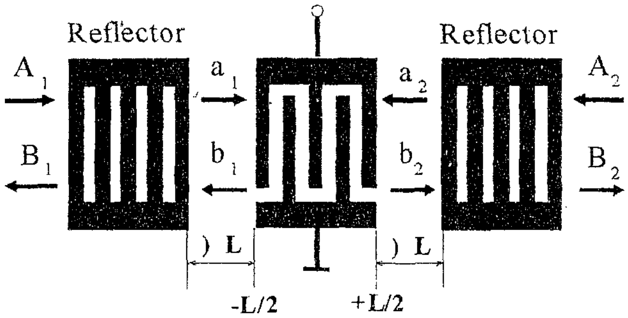

Fig. 4. Basic layout of the one-port SAW resonator

A lossless SAW transducer is characterized by the mixed scattering matrix

$$\begin{bmatrix} b_1 \\ b_2 \\ I \end{bmatrix}=\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ V \end{bmatrix}=\begin{bmatrix} r e^{-j\beta L} & t e^{-j\beta L} & m e^{-j\beta L/2} \\ t e^{-j\beta L} & r e^{-j\beta L} & m^* e^{-j\beta L/2} \\ -2m e^{-j\beta L/2} & -2m^* e^{-j\beta L/2} & Y_0 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ V \end{bmatrix} \tag{15}$$

as referenced to the acoustic ports with the coordinates $\pm L/2$ where $\beta=\omega/v$ is the wave number In Eq (15) r and t denote the reflection and transduction coefficients, respectively, m is the electroacoustic transfer function, and Y_0 is the transducer admittance

The gaps of the length ΔL between the central transducer and reflective gratings can be modeled as the transmission lines with the transmission matrix

$$\mathbf{T}=\begin{bmatrix} e^{j\beta_0 \Delta L} & 0 \\ 0 & e^{j\beta_0 \Delta L} \end{bmatrix} \tag{16}$$

where $\beta_0=\omega/v_0$ is the wave number for the free substrate surface For simplicity, we assume that transmission lines (16) in the gaps are cascaded with transmission matrices of the reflectors and hence the SAW system is reduced to the 3-port SAW transducer loaded by two two-port junctions

Rewrite Eq. (15) in the block-matrix form

$$\begin{bmatrix} \mathbf{b} \\ I \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{aa} & \mathbf{M}_{ae} \\ \mathbf{M}_{ea} & \mathbf{M}_{ee} \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ V \end{bmatrix} \tag{17}$$

where the matrix blocks are defined as follows

$$\mathbf{M}_{aa} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}, \quad \mathbf{M}_{ae} = \begin{bmatrix} m_{13} \\ m_{23} \end{bmatrix}, \quad \mathbf{M}_{ea} = [m_{31} \quad m_{32}]; \quad \mathbf{M}_{ee} = m_{33}$$

In the block-matrix representation (17) the index "a" is attributed to the acoustic terms and the index "e" is attributed to the electric terms Thus, the total matrix contains one acoustic block $\mathbf{M}_{aa}$; electroacoustic blocks $\mathbf{M}_{ea}=-2\mathbf{M}_{ae}^T$, and one scalar electric term $\mathbf{M}_{ee}$.

By applying the transmission relation

$$\begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{11} & \mathbf{T}_{12} \\ \mathbf{T}_{21} & \mathbf{T}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{A} \\ \mathbf{B} \end{bmatrix} \tag{18}$$

we obtain the mixed scattering matrix of a SAW resonator

$$\begin{bmatrix} \mathbf{B} \\ I \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{A} \\ V \end{bmatrix} \tag{19}$$

where

$$\begin{aligned} &\mathbf{M}_{11} = (\mathbf{T}_{22} - \mathbf{M}_{aa}\mathbf{T}_{12})^{-1}(\mathbf{M}_{aa}\mathbf{T}_{11} - \mathbf{T}_{21}); \quad \mathbf{M}_{12} = (\mathbf{T}_{22} - \mathbf{M}_{aa}\mathbf{T}_{12})^{-1}\mathbf{M}_{ae}; \\ &\mathbf{M}_{21} = \mathbf{M}_{ea}\left[\mathbf{T}_{11} + \mathbf{T}_{12}(\mathbf{T}_{22} - \mathbf{M}_{aa}\mathbf{T}_{12})^{-1}(\mathbf{M}_{aa}\mathbf{T}_{11} - \mathbf{T}_{21})\right]; \\ &\mathbf{M}_{22} = \mathbf{M}_{ee} + \mathbf{M}_{ea}\mathbf{T}_{12}(\mathbf{T}_{22} - \mathbf{M}_{aa}\mathbf{T}_{12})^{-1}\mathbf{M}_{ae} \end{aligned} \tag{20}$$

Finally, using the Eqs. (19-20) the one-port resonator admittance $Y=I/V=\mathbf{M}_{22}$ can be found in the matrix form as

$$Y = Y_0 + \mathbf{M}_{ea}\mathbf{T}_{12}(\mathbf{T}_{22} - \mathbf{M}_{aa}\mathbf{T}_{12})^{-1}\mathbf{M}_{ae} = Y_0 - 2\mathbf{M}_{ae}^T\mathbf{T}_{12}(\mathbf{T}_{22} - \mathbf{M}_{aa}\mathbf{T}_{12})^{-1}\mathbf{M}_{ae} \tag{21}$$

where Y_0 is the admittance of the SAW transducer

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