Charge Distribution and Capacitance Calculation for Generalized Periodic SAW Transducers Using Floquet's Technique

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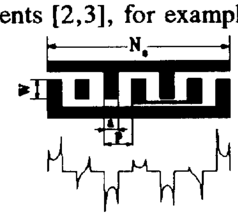
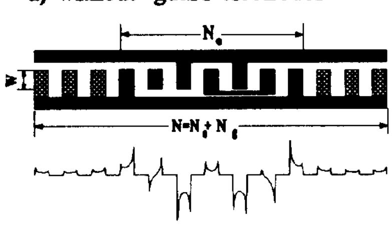
Abstract — Analytic formulae for the charge density, total charges on the electrodes, and static capacitance of the generalized periodic SAW transducers are derived in the paper using the basic analytic solution for a periodic phased array transducer. Static capacitance is determined by summation of the weighted interelectrode capacitors, with the capacitors values and weights found analytically. The solution of the mixed electrostatic problem where each electrode might be characterized either by its voltage or by its charge is also considered. As a special case, the solution for transducers with separate or interconnected floating strips follows from the general solution.

INTRODUCTION

Properties of SAW interdigital transducers (IDT) can be deduced, to the first order, from the electrostatic solution ignoring piezoelectricity (quasi-static approximation) [1]. Rigorous treatments of the problem for arbitrary finite length IDT based on the Green's function approach are available [1-3]. However, extensive calculations are necessary to evaluate charge density distribution and/or IDT capacitance by applying point-matching techniques (method of moments [2,3], for example).



a) without guard electrodes

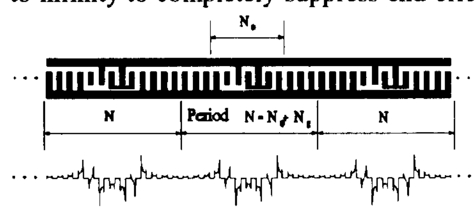


b) with guard electrodes

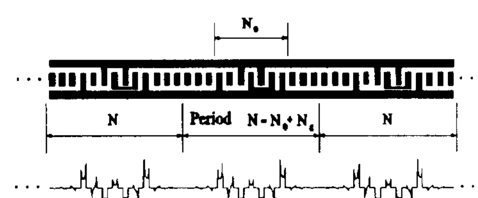
Fig. 1. Finite length SAW transducer

Fortunately, many practical IDT can be modeled as a periodic array of metallic strips with arbitrary voltages (Fig. 1a). The solution is considerably simplified by applying superposition principle and the known analytic solution for the elemental charge density distribution in the infinite periodic single tap transducer having all electrodes grounded except for the one with unity potential [4]. End effects associated with charge density distortion near the ends of a real finite length IDT may be suppressed by adding special dummy (guard) grounded fingers at each side of the IDT [4] (Fig. 1b).

However, known solutions in terms of the elemental charge density distribution in the infinite single tap transducer [5,6] suffer from some drawbacks. In general case the solution includes complicated integrals of Legendre functions that is inconvenient for practical use. Moreover, there is uncertainty in the number of guard electrodes to be introduced in a real IDT as this number depends on the transducer geometry and tends theoretically to infinity to completely suppress end effects.



a) connected (grounded) uncoupling electrodes



b) disconnected (floating) uncoupling electrodes

Fig. 2. Hypothetic generalized periodic SAW transducer

Another approach to modeling periodic SAW transducers is proposed in the present paper. The initial electrostatic problem is approximated by an auxiliary one with the periodic boundary conditions on the surface. To this

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end, an entire IDT containing N electrodes with arbitrary voltages V_i is treated as one generalized period of the infinite periodic array derived by sequential multiple repetition of the initial transducer (Fig.2). Provided for sufficient uncoupling between adjacent periods due to special uncoupling grounded (Fig. 2a) or floating (Fig. 2b) fingers, the solution to be obtained for one period might be a good approximation to the initial problem.

Assumed for all the electrode voltages to be a priori prescribed, the closed-form solution of the auxiliary electrostatic problem can be deduced from the basic charge density distribution in a periodic phased array of the strips with the same voltages impressed and phase progressing uniformly along the array [5] by applying Floquet's theorem and superposition principle. Results for infinite periodic arrays [5,6] follow from the theory as a particular limiting case of the infinite period $N \to \infty$.

THEORY AND APPLICATIONS

Statement of the problem

The initial electrostatic problem is posed by the electrode geometry of Fig. 1 where the basic periodic structure of N_0 strips is considered. All the strips have uniform width a and fixed pitch p throughout the transducer with a constant metallization ratio $\eta = a/p$. Electrodes are allowed to take on any voltages V_i with arbitrary magnitude and phase. It is assumed where appropriate that electrodes are concerned to either of two bus-bars, some of them being disconnected (floating) ones with the unknown a priori voltages in general case. The floating electrodes may be separate or interconnected (sectioned).

An auxiliary periodic structure with connected (grounded) or disconnected (floating) uncoupling fingers is shown in Fig. 2. It is worthy noting that floating uncoupling strips in Fig. 2b allow better uncoupling between adjacent periods. However, their presence complicates considerably the problem as their potentials must be beforehand calculated as well as the unknown potentials on the floating strips in the basic structure if available.

The problem is to determine charge density distribution, electrode charges, unknown voltages, and capacitance for one generalized period containing $N = N_0 + N_g$ electrodes, with N_0 being an electrode number in the basic structure and N_g being an uncoupling (guard) electrode number.

Phased array transducer and basic analytic relations

According to the Floquet's theorem for periodic structures voltages and charges on the same electrodes of different periods are the same apart from the phase shift. Therefore, the electrostatic problem should be solved for one generalized period only.

Let us suppose for the moment that all the electrode voltages are known and an arbitrary voltage sequence on the electrodes can be synthesized as follows

$$V_i = \frac{1}{N}\sum_{s=0}^{N-1} \tilde{V}_s e^{-j\varphi_s i}, \quad \tilde{V}_s = \sum_{i=0}^{N-1} V_i e^{j\varphi_i s}, \quad \varphi_s = 2\pi s/N \qquad (1)$$

or in the matrix form

$$\mathbf{V} = \mathbf{H}\tilde{\mathbf{V}}, \qquad \tilde{\mathbf{V}} = \mathbf{H}^{-1}\mathbf{V} \qquad (2)$$

where $\mathbf{V} = [V_0\ V_1\ \ldots V_{N-1}]^T$ is the vector of the electrode voltages and an auxiliary vector $\tilde{\mathbf{V}} = [\tilde{V}_0\ \tilde{V}_1\ \ldots \tilde{V}_{N-1}]^T$ contains voltages on a set of the phased array transducers, each with the same strip voltage $\tilde{V}_s$ and phase progressing uniformly along the array at a rate $\varphi_s = 2\pi s/N$ (Fig. 3).

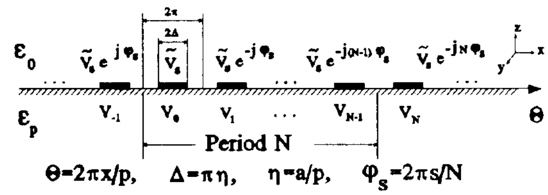

Fig. 3. Phased array transducer

The vectors $\mathbf{V}$ and $\tilde{\mathbf{V}}$ are interrelated via a square matrix $\mathbf{H} = [h_{is}]_{N-1}$, $h_{is} = 1/N \exp\{-j\varphi_s i\}$. The matrix $\mathbf{H}$ has the closed-form inverse matrix $\mathbf{H}^{-1} = N\ \mathbf{H}^*$ where the asterisk denotes matrix Hermitian conjugation.

The charge distribution in a phased array transducer is known to be of the form [5,6]

$$\tilde{\sigma}_s(\Theta) = \tilde{\gamma}_s(\Theta)\tilde{V}_s,$$

$$\tilde{\gamma}_s(\Theta) = \frac{2\sqrt{2}\,\varepsilon}{p}\,\frac{\sin\pi s/N}{P_{-s/N}(-\cos\Delta)}\,\frac{e^{-j(s/N-\frac{1}{2})\Theta}}{\sqrt{\cos\Delta - \cos\Theta}}, \quad |\Theta| \le \Delta \qquad (3)$$

where $\Delta = \pi\eta$; $\varepsilon = \varepsilon_0 + \varepsilon_p$ is effective permittivity of the surface, ε_0 is permittivity of the medium above the surface; $\varepsilon_p = (\varepsilon_{11}\varepsilon_{33} - \varepsilon_{13}^2)^{1/2}$ is permittivity of the substrate; $P_{-s/N}(-\cos\Delta)$ is Legendre function, and Θ denotes a dimensionless variable related to coordinate x by $\Theta = 2\pi x/p$.

The total charge on the electrode of a phased array transducer can be found by integrating (3) over the strip width [5,6] using Mehler-Dirichlet's formula [7]

$$\tilde{Q}_s = \tilde{\gamma}_s\,\tilde{V}_s, \quad \tilde{\gamma}_s = 2\varepsilon\sin\pi s/N\,\frac{P_{-s/N}(\cos\Delta)}{P_{-s/N}(-\cos\Delta)} \qquad (4)$$

Thus, an arbitrary periodic IDT containing N electrodes in a generalized period can be effectively modeled by superposition of a set of N phased array transducers with the known closed-form solutions (3),(4) for charge distribution in a separate phased array transducer.

Surface charge density distribution

By applying superposition principle the charge density distribution on the i-th electrode can be expressed as

$$\sigma_i(\Theta)=\frac{1}{N}\sum_{s=0}^{N-1}\tilde{\sigma}_s(\Theta)\,e^{-j\varphi_s i} \tag{5}$$

where $\tilde{\sigma}_s(\Theta)$ is the charge density distribution in the s-th phased array transducer with the strip voltage $\tilde{V}_s$. By substituting (3) into (5) and taking into account the second Eq. (1) we can derive after some manipulations with indexes the following expression

$$\sigma_i(\Theta)=\sum_{p=0}^{N-1}\gamma_p(\Theta)\,V_{i-p},\quad \gamma_p(\Theta)=\frac{1}{N}\sum_{s=0}^{N-1}\tilde{\gamma}_s(\Theta)\,e^{-j2\pi ps/N} \tag{6}$$

where coefficients $\gamma_p(\Theta)$ represent the basic charge density distribution on the period of N electrodes with one electrode activated and all others grounded (the zero index $p=0$ is attributed to the activated finger). According to Eq. (6) the charge distribution $\sigma_i(\Theta)$ is given by the convolution product of the basic charge distribution $\gamma_p(\Theta)$ with the electrode voltages V_i.

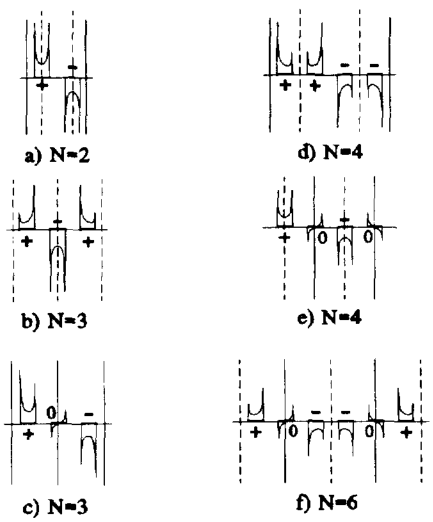

Fig. 4. Charge density distribution on the period of multielectrode periodic Engan's transducers

As an example the charge density distributions on the periods of multielectrode periodic Engan's transducers [8] are shown in Fig. 4 for N=2,3,4 and 6 with different polarity sequences and metallization ratio $\eta=0.5$.

Interrelation of the electrode charges and voltages. Capacitance and potential matrices

By integrating (6) over the electrode width we can deduce the following closed-form relations to express charges in terms of voltages and vice versa:

$$Q_i=\sum_{p=0}^{N-1}\gamma_p V_{i-p},\qquad V_i=\sum_{p=0}^{N-1}\gamma_p^{+} Q_{i-p} \tag{7}$$

$$\begin{matrix}\gamma_p\\ \gamma_p^{+}\end{matrix}=\frac{1}{N}\sum_{s=0}^{N-1}\begin{matrix}\tilde{\gamma}_s\\ \tilde{\gamma}_s^{+}\end{matrix}\;e^{-j2\pi ps/N} \tag{8}$$

or in the matrix form

$$\mathbf{Q}=\mathbf{\Gamma}\,\mathbf{V},\qquad \mathbf{V}=\mathbf{\Gamma}^{+}\mathbf{Q} \tag{9}$$

$$\mathbf{\Gamma}=\mathbf{H}^{-1}\tilde{\mathbf{\Gamma}}\,\mathbf{H},\qquad \mathbf{\Gamma}^{+}=\mathbf{H}^{-1}\tilde{\mathbf{\Gamma}}^{+}\mathbf{H} \tag{10}$$

where "+" denotes pseudo-inversion [9] of the matrix $\mathbf{\Gamma}$ which is degenerate due to the charge neutrality condition. The elements of the diagonal matrices $\tilde{\mathbf{\Gamma}}$ and $\tilde{\mathbf{\Gamma}}^{+}$ are interrelated as $\tilde{\gamma}_s^{+}=1/\tilde{\gamma}_s$, $s\neq 0$ with $\tilde{\gamma}_0^{+}=0$ where the matrix elements $\tilde{\gamma}_s$ are given by (4). The capacitance matrix $\mathbf{\Gamma}$ relating charges to voltages has the structure

$$\mathbf{\Gamma}=\begin{bmatrix}\gamma_0 & \gamma_1 & \gamma_2 & \cdots & \gamma_2 & \gamma_1\\ \gamma_1 & \gamma_0 & \gamma_1 & \cdots & \gamma_3 & \gamma_2\\ \gamma_2 & \gamma_1 & \gamma_0 & \cdots & \gamma_4 & \gamma_3\\ \vdots & & & & & \vdots\\ \gamma_2 & \gamma_3 & \gamma_4 & \cdots & \gamma_0 & \gamma_1\\ \gamma_1 & \gamma_2 & \gamma_3 & \cdots & \gamma_1 & \gamma_0\end{bmatrix} \tag{11}$$

and the pseudo-inverse potential matrix $\mathbf{\Gamma}^{+}$ relating voltages to charges has the same structure.

Quantities $\gamma_p=\gamma_p(N,\eta)$ are charges on the electrodes of the basic periodic structure with the only electrode of the period activated. They are slowly varying functions of the metallization ratio η and fall off rapidly (at a rate of the order $1/p^2$) as the relative electrode number p increases. The coefficients γ_p have the simplest form for a special case of $\eta=0.5$ when $\tilde{\gamma}_s=2\varepsilon\sin\pi s/N$. Then Eq. (8) may be convolved to the simple analytic expression

$$\gamma_p=2\epsilon\,\frac{\sin p/N}{N(\cos 2\pi p/N-\cos p/N)},\quad p=0,1,\ldots,N-1. \tag{12}$$

The known result [5] follows from Eqs. (4) and (8) as a particular limiting case of $N\rightarrow\infty$, with a discrete variable s/N replaced by a continuous variable ν and summation replaced by integration:

$$\gamma_p=\int_0^1\tilde{\gamma}(\nu)\,e^{-j2\pi p\nu}\,d\nu,\quad \tilde{\gamma}(\nu)=2\epsilon\,\sin\pi\nu\;\frac{P_{-\nu}(\cos\Delta)}{P_{-\nu}(-\cos\Delta)},\quad p=0,\ 1,\ 2,\ \ldots \tag{13}$$

Static capacitance of SAW transducers

We can deduce an analytic expression for the static capacitance C using the energy conservation law written in the form

$$\tfrac{1}{2}C\Delta V^2=\tfrac{1}{2}\mathbf{V}^{*}\mathbf{Q} \tag{14}$$

where ΔV is voltage applied to the transducer bus-bars, V and Q are vectors of the voltages and charges on the electrodes. Substituting the first Eq. (9) into (14) leads to the following expressions for the capacitance of an unapodized SAW transducer of the unit aperture $W=1$

$$C=\frac{1}{\Delta V^2}\mathbf{V}^{*}\mathbf{\Gamma}\mathbf{V}=\frac{1}{N\Delta V^2}\tilde{\mathbf{V}}^{*}\tilde{\mathbf{\Gamma}}\tilde{\mathbf{V}} \tag{15}$$

or in the scalar form

$$C=\frac{1}{\Delta V^2}\sum_{i=0}^{N-1}\sum_{k=0}^{N-1}\gamma_{ik}V_i V_k=\frac{1}{N}\sum_{s=1}^{N-1}\tilde{C}_s,\quad \tilde{C}_s=\tilde{\gamma}_s\left|\frac{\tilde{V}_s}{\Delta V}\right|^2 \tag{16}$$

where quantities $\gamma_{ik}=\gamma_{|i-k|}$, $i\neq k$ may be interpreted as the charges on the capacitors $C_p=-\gamma_p$, $p=|i-k|$ between i-th and k-th electrodes (Fig. 5) and γ_0 is equal to the total sum of the charges on all the capacitors.

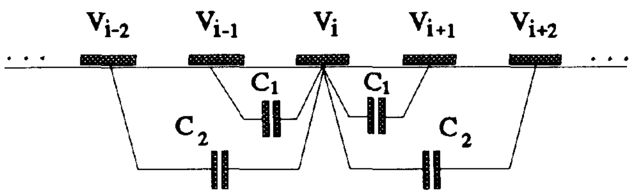

Fig. 5. Interelectrode capacitors

Using the identities

$$2V_iV_k=V_i^2+V_k^2-(V_i-V_k)^2,\quad \sum_{i=0}^{N-1}\gamma_{ik}=\sum_{k=0}^{N-1}\gamma_{ik}=0 \tag{17}$$

and taking into account the full transducer aperture W formula (16) may be converted to the form

$$C=-\frac{1}{2}\sum_{i=0}^{N-1}\sum_{k=0}^{N-1}\gamma_{ik}W_{ik}=-\sum_{i=0}^{N-1}\sum_{k=0}^{i-1}\gamma_{ik}W_{ik},\quad W_{ik}=\left(\frac{V_i-V_k}{\Delta V}\right)^2 W \tag{18}$$

If each electrode is connected to either of bus-burs ($V_i=\pm\Delta V/2$) then $W_{ik}=0$ for $V_i=V_k$ and $W_{ik}=W$ for $V_i\neq V_k$. Thus, the partial apertures W_{ik} are equal to the overlaps of the i-th and k-th electrodes.

The expression (18) is converted to the compact form by using symmetry properties and reordering summation

$$C=\sum_{p=1}^{N-1}C_pL_p,\quad L_p=\sum_{k=0}^{N-1-p}W_{k,k+p} \tag{19}$$

where quantities L_p are total overlaps of all the nearest, next nearest neighbour electrodes, and so on respectively. Therefore, static capacitance is composed of the weighted interelectrode capacitors, with formula (19) being applicable both to unapodized and apodized SAW transducers with arbitrary polarity sequences.

The known Engan's results for the capacitance of multielectrode periodic transducers [8] follow from Eq. (16) as a special case for N=2,3,4 and 6, capacitance values per period shown in Fig. 6 for metallization ratio $\eta=0.5$.

N	V_k	$\tilde{\gamma}_1/\epsilon$	$\lvert\tilde{V}_1/\Delta V\rvert^2$	C/ϵ
2	+ -	2	1	1
3	+ - +	$\sqrt{3}$	1	$2/\sqrt{3}$
	+ 0 -	$\sqrt{3}$	3/4	$\sqrt{3}$
4	+ + - -	$\sqrt{2}$	2	$\sqrt{3}/2$
	+ 0 - 0	$\sqrt{2}$	1	$1/\sqrt{2}$
6	+0--0+	1	3	1

Fig. 6. Capacitance per period for multielectrode periodic Engan's transducers

Mixed electrostatic problem solution

Given a priori all the electrode voltages, the charges on the electrodes can be calculated from the first Eq. (9) using the capacitance matrix $\mathbf{\Gamma}$ and vice versa if all the electrode charges are known the voltages can be recovered from the second Eq. (9) using the pseudo-inverse potential matrix $\mathbf{\Gamma}^{+}$.

Now consider more general mixed electrostatic problem where each electrode may be characterized either by its potential or by its charge prescribed a priori.

Rewrite the first Eq. (9) in the block-matrix form

$$\begin{bmatrix}\mathbf{Q}_0\\ \mathbf{Q}_1\end{bmatrix}=\begin{bmatrix}\mathbf{\Gamma}_{00} & \mathbf{\Gamma}_{01}\\ \mathbf{\Gamma}_{10} & \mathbf{\Gamma}_{11}\end{bmatrix}\begin{bmatrix}\mathbf{V}_0\\ \mathbf{V}_1\end{bmatrix} \tag{20}$$

where $\mathbf{V}_0$, $\mathbf{Q}_1$ are vectors of the prescribed voltages and charges and $\mathbf{Q}_0$, $\mathbf{V}_1$ are vectors of the unknown charges and voltages. It is assumed in (20) that vector and matrix elements are properly permutated and renumbered.

The solution of the matrix equations (20) with respect to the unknown vectors $\mathbf{Q}_0$ and $\mathbf{V}_1$ is given by

$$\begin{bmatrix}\mathbf{Q}_0\\ \mathbf{V}_1\end{bmatrix}=\begin{bmatrix}\mathbf{\Gamma}_{00}-\mathbf{\Gamma}_{01}\mathbf{\Gamma}_{11}^{-1}\mathbf{\Gamma}_{10} & \mathbf{\Gamma}_{01}\mathbf{\Gamma}_{11}^{-1}\\ -\mathbf{\Gamma}_{11}^{-1}\mathbf{\Gamma}_{10} & \mathbf{\Gamma}_{11}^{-1}\end{bmatrix}\begin{bmatrix}\mathbf{V}_0\\ \mathbf{Q}_1\end{bmatrix} \tag{21}$$

where the vectors $\mathbf{Q}_0$ and $\mathbf{V}_1$ are expressed in terms of the a priori prescribed vectors $\mathbf{V}_0$ and $\mathbf{Q}_1$. An iterative Gauss-Seidel procedure may be applied instead of the matrix $\mathbf{\Gamma}_{11}$ inversion, with the storage requirements considerably reduced as only $N/2$ different coefficients γ_p should be stored.

SAW transducers with floating strips

The solution for SAW transducers with separate or interconnected (sectioned) floating strips may be treated as a special case of the mixed electrostatic problem, with the charge neutrality and equipotentiality conditions imposed to the floating sections.

Rewrite the first Eq. (9) in the partitioned form

$$\begin{bmatrix} \mathbf{Q}_0 \\ \mathbf{Q}_1 \\ \vdots \\ \mathbf{Q}_M \end{bmatrix} = \begin{bmatrix} \Gamma_{00} & \Gamma_{01} & \cdots & \Gamma_{0M} \\ \Gamma_{10} & \Gamma_{11} & \cdots & \Gamma_{1M} \\ \vdots & & & \vdots \\ \Gamma_{M0} & \Gamma_{M1} & \cdots & \Gamma_{MM} \end{bmatrix} \begin{bmatrix} \mathbf{V}_0 \\ \mathbf{V}_1 \\ \vdots \\ \mathbf{V}_M \end{bmatrix} \quad (22)$$

where $\mathbf{Q}_0$ and $\mathbf{V}_0$ denote vectors of charges and voltages on the connected fingers and subvectors $\mathbf{Q}_i$ and $\mathbf{V}_i$, $i=\overline{1, M}$ with elements Q_k^i and V_k^i, $k=\overline{1, N_i}$ contain charges and voltages on the strips of the floating sections, with M being a total number of the floating sections including separate floating strips and N_i being a number of the strips in the i-th section.

Apply to each floating section the charge neutrality and equipotentiality conditions:

$$\mathbf{L}_i^T \mathbf{Q}_i = 0 \quad \rightarrow \quad \sum_{k=1}^{N_i} Q_k^i = 0, \quad i=\overline{1, M} \quad (23)$$

$$\Phi_i \mathbf{L}_i = \mathbf{V}_i \quad \rightarrow \quad V_k^i = \Phi_i, \quad i=\overline{1, M}, \quad k=\overline{1, N_i} \quad (24)$$

where $\mathbf{\Phi}=[\Phi_1\ \Phi_2\ \ldots\ \Phi_M]^T$ is the vector of the unknown voltages on the floating sections and the vectors $\mathbf{L}_i=[1\ 1\ \ldots\ 1]^T$ contain N_i unity elements. Then Eq. (22) may be convolved to the reduced form

$$\begin{bmatrix} \mathbf{Q}_0 \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathfrak{S}_{00} & \mathfrak{S}_{01} \\ \mathfrak{S}_{10} & \mathfrak{S}_{11} \end{bmatrix} \begin{bmatrix} \mathbf{V}_0 \\ \mathbf{\Phi} \end{bmatrix} \quad (25)$$

where $\mathfrak{S}_{ik}$ are the appropriate matrix-blocks of the reduced capacitance matrix $\mathfrak{S}$ and the zero vector $\mathbf{0}=[0\ 0\ \ldots\ 0]^T$ of length M contains zero charges on the floating sections. Solving the second Eq. (25) with respect to the unknown vector $\mathbf{\Phi}$ leads to

$$\mathbf{\Phi} = -\mathfrak{S}_{11}^{-1} \mathfrak{S}_{10} \mathbf{V}_0 \quad (26)$$

where the unknown voltages Φ_i are expressed in terms of the prescribed voltages V_k on the connected fingers.

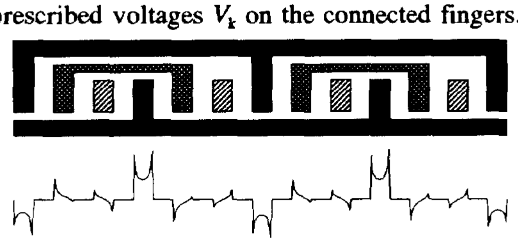

Fig. 7. Hypothetic generalized SAW transducer with floating strips

As an example the charge density distribution on the electrodes of the hypothetic generalized SAW transducer comprising both separate and interconnected floating strips is shown in Fig. 7., with the unknown potentials on the floating strips calculated using Eq. (26). The results of calculations are in excellent agreement with those obtained by the numeric method of moments [3].

CONCLUSION

The complete closed-form solution of the electrostatic problem for periodic SAW transducers has been considered in this paper where IDT is treated as one generalized period of the periodic structure derived by sequential multiple repetition of the initial IDT. The transducer may contain separate or interconnected floating strips. Special uncoupling grounded or disconnected fingers are introduced to uncouple between adjacent periods while calculating and to suppress end effects in a finite length IDT.

Known results for infinite periodic arrays follow from the theory as a particular limiting case of the infinite period. However, the advantages of the proposed approach are apparent:

1. Physical redundancy of the infinite array is removed.
2. There is a simple criterion (uncoupling between periods) to evaluate a number of the uncoupling fingers to be introduced in a real IDT to suppress end effects.
3. Closed-form expressions are derived in terms of finite summation instead of finite series and integrals.

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