Closed-form solution of the electrostatic problem for generalized periodic SAW transducers

A.S.Rukhlenko

Dept. of Semiconductor Physics, Belarusian State University, 4, Fr. Skorina Ave., Minsk, 220050, Republic of Belarus

ABSTRACT

Analytic formulae for charge density distribution, total charges on electrodes, and static capacitance of generalized SAW periodic transducers with arbitrary polarity sequences are derived in the paper using Floquet's theorem and superposition principle. The electrode charge density is expressed in terms of the basic analytic solution for a periodic phased array of the strips with the same voltages applied and the phase progressing uniformly along the array. The closed-form capacitance matrix and pseudo-inverse potential matrix interrelating electrode voltages and charges within a generalized period of the transducer are derived. Transducer static capacitance is determined by appropriate summation of the interelectrode capacitors, with the capacitors values given by the capacitance matrix elements. The solution of the mixed electrostatic problem where each electrode might be characterized either by its potential or its charge is considered. The vectors of the unknown charges and voltages are expressed in terms of the vectors of the a priori prescribed voltages and charges. As a special case, the solution for transducers with separate or interconnected (sectioned) floating strips follows from the general solution.

1. INTRODUCTION

Properties of SAW interdigital transducers can be deduced, to the first order, from the electrostatic solution ignoring piezoelectricity (quasi-static approximation)[1]. Rigorous treatments of the problem for arbitrary finite length transducers based on the one- or two-dimensional Green's function approach are available[2-5]. However, extensive calculations are necessary to evaluate charge density distribution and/or transducer capacitance by applying point-matching techniques (method of moments, for example) to numerically solve integral equations involved.

Fortunately, many practical transducers can be modeled as a periodic array of metallic strips with arbitrary voltages impressed (Fig. 1a). The solution is considerably simplified by applying superposition principle and the known analytic solution for the elemental charge density distribution in an infinite periodic single tap transducer having all electrodes grounded except for the one with unity potential[6,7]. End effects associated with charge density distortion near the ends of a real finite length transducer may be rather effectively suppressed by adding some special dummy (guard) grounded fingers at each side of the real transducer[6,7] (Fig.1b).

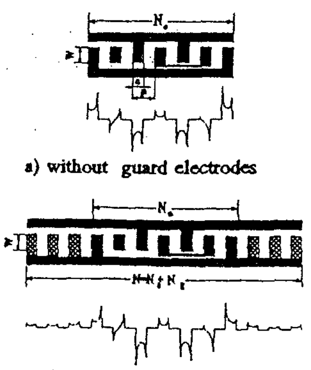

Fig.1. Finite length SAW transducer and surface charge density distribution on the electrodes

However, known solutions in terms of the elemental charge density distribution in the infinite single tap transducer[8,9] suffer from some drawbacks. In general case the solution includes the complicated integrals of Legendre functions that is inconvenient for practical use. Moreover, there is uncertainty in the number of guard electrodes to be introduced in the real transducer. This number depends on the transducer geometry and tends theoretically to infinity to completely suppress end effects.

Another approach to modeling periodic SAW transducers is proposed in the present paper. The initial electrostatic problem is approximated by an auxiliary one with the periodic boundary conditions on the surface. To this end, an entire transducer containing N electrodes with arbitrary voltages V_i is treated as one generalized period of the infinite periodic array derived by sequential multiple repetition of the initial transducer[10] (Fig 2). Provided for sufficient uncoupling between adjacent periods due to the special uncoupling grounded (Fig 2a) or floating (Fig 2b) electrodes, the solution to be obtained for one period might be a good approximation to the initial problem. As it will be shown the closed-form solution of the auxiliary electrostatic problem can be deduced from the basic charge density distribution in a periodic phased array of the strips with the same voltages impressed and the phase progressing uniformly along the array by applying Floquet's theorem and superposition principle, with all the electrode voltages supposed to be a priori prescribed or calculated. Previous results for infinite periodic SAW transducers[8,9] follow from the theory as a particular limiting case of the infinite period $N \to \infty$

2. THEORY AND APPLICATIONS

2.1. Statement of the problem

The initial electrostatic problem is posed by the electrode geometry of Fig. 1 where the basic periodic structure of N_0 strips is considered. All the strips have uniform width a and fixed pitch p throughout the transducer with a constant metallization ratio $\eta=a/p$. Electrodes are allowed to take on any voltages V_i with arbitrary magnitude and phase. It is assumed where appropriate that electrodes are concerned to either of two bus-bars, some of them being disconnected (floating) with the unknown a priori voltages in general case. The floating electrodes may be separate or interconnected (sectioned).

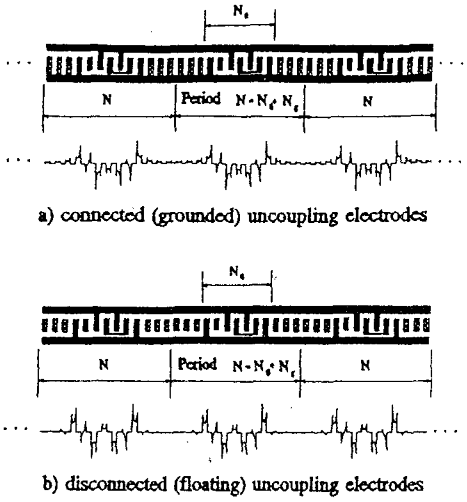

Fig. 2. Hypothetic generalized periodic SAW transducer with uncoupling electrodes between periods

An auxiliary periodic structure with connected (grounded) or disconnected (floating) uncoupling electrodes is shown in Fig. 2. It is worthy noting that the floating uncoupling electrodes (Fig. 2b) allow better uncoupling between periods if compared to the grounded ones (Fig. 2a). However, their presence complicates considerably the problem as their unknown potentials must be beforehand calculated as well as the potentials on the floating electrodes in the basic structure if available. Thus, one generalized period contains $N=N_0+N_g$ electrodes altogether in general case, with N_0 being an electrode number in the basic structure and N_g being an uncoupling (guard) electrode number. The problem is to determine charge density distribution, total charges, unknown voltages, and static capacitance for a generalized period.

2.2. Phased array transducer and basic analytic relations

According to the Floquet's theorem for periodic structures voltages and charges on the same electrodes of different periods are the same apart from the phase shift. Therefore, the electrostatic problem should be solved for one generalized period only.

Let us suppose for the moment that all the electrode voltages are known and an arbitrary voltage sequence on the electrodes can be synthesized as follows

$$V_i=\frac{1}{N}\sum_{s=0}^{N-1}\tilde{V}_s e^{-j\varphi_s i} \quad where \quad \tilde{V}_s=\sum_{i=0}^{N-1}V_i e^{j\varphi_i s}, \quad \varphi_s=2\pi s/N \tag{1}$$

or in the matrix form

$$V=H\tilde{V}, \qquad \tilde{V}=H^{-1}V \tag{2}$$

where $\mathrm{V}=[V_0\ V_1\ \ldots\ V_{N-1}]^T$ is the vector of the transducer electrode voltages and an auxiliary vector $\tilde{\mathrm{V}}=[\tilde{V}_0\ \tilde{V}_1\ \ldots\ \tilde{V}_{N-1}]^T$ contains voltages on a set of the phased array transducers, each with the same voltage $\tilde{V}_s$ applied to the strips and the phase progressing uniformly along the array at a rate $\varphi_s=2\pi s/N$ (Fig. 3). The vectors V and $\tilde{\mathrm{V}}$ are interrelated via a square matrix $\mathrm{H}=[h_{is}]_{N-1}$ having the closed-form inverse matrix $\mathrm{H}^{-1}=N\,\mathrm{H}^*$ where the asterisk denotes Hermitian conjugation, with the matrix H elements given by $h_{is}=1/N\exp\{-j\varphi_s i\}$.

The charge distribution in a phased array transducer is known to be of the form[8,9]

$$\tilde{\sigma}_s(\Theta)=\tilde{\gamma}_s(\Theta)\,\tilde{V}_s, \qquad \tilde{\gamma}_s(\Theta)=\frac{2\sqrt{2}\,\varepsilon}{p}\,\frac{\sin\pi s/N}{P_{-s/N}(-\cos\Delta)}\,\frac{e^{-j(s/N-\frac{1}{2})\Theta}}{\sqrt{\cos\Delta-\cos\Theta}}, \qquad |\Theta|\le\Delta \tag{3}$$

where $\Delta=\pi\eta$; $\varepsilon=\varepsilon_0+\varepsilon_p$ is effective permittivity of the surface, with ε_0 being permittivity of the medium above the surface and $\varepsilon_p=(\varepsilon_{11}\varepsilon_{33}-\varepsilon_{13}^2)^{1/2}$ being permittivity of the substrate; $P_{-s/N}(-\cos\Delta)$ is Legendre function, and Θ denotes a dimensionless variable related to coordinate x by $\Theta=2\pi x/p$.

The total charge on the electrode of the phased array transducer can be found by integrating (3) over the electrode width[8,9] using Mehler-Dirichlet's formula[11]

$$\tilde{Q}_s=\tilde{\gamma}_s\tilde{V}_s, \qquad \tilde{\gamma}_s=2\varepsilon\sin\pi s/N\,\frac{P_{-s/N}(\cos\Delta)}{P_{-s/N}(-\cos\Delta)} \tag{4}$$

Thus, an arbitrary periodic SAW transducer containing N electrodes in a generalized period may be effectively modeled

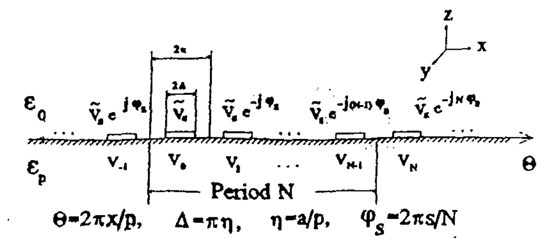

Fig. 3. Phased array transducer

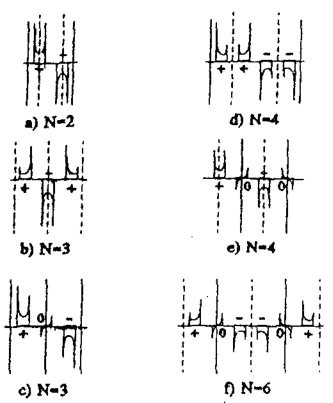

Fig. 4. Charge density distribution on the period of the multielectrode periodic Engan's transducers

as an appropriate superposition of a set of N phased array transducers with the known closed-form solutions (3),(4) for charge distribution in a separate phased array transducer.

2.3. Surface charge density distribution

By applying superposition principle the charge density distribution on the i-th electrode can be expressed as

$$\sigma_i(\Theta)=\frac{1}{N}\sum_{s=0}^{N-1}\tilde{\sigma}_s(\Theta)\,e^{-j\varphi_s i} \tag{5}$$

where $\tilde{\sigma}_s(\Theta)$ is the charge density distribution in the s-th phased array transducer with the strip voltage $\tilde{V}_s$. By substitution (3) into (5) and taking into account the second Eq. (1) we can derive after some manipulations with indexes the following expression

$$\sigma_i(\Theta)=\sum_{p=0}^{N-1}\gamma_p(\Theta)\,V_{i-p},\qquad \gamma_p(\Theta)=\frac{1}{N}\sum_{s=0}^{N-1}\tilde{\gamma}_s(\Theta)\,e^{-j2\pi ps/N} \tag{6}$$

where coefficients $\gamma_p(\Theta)$ represent the basic charge density distribution on the period of N electrodes with one electrode activated and all others grounded (the zero index $p=0$ is attributed to the activated strip). According to (6) charge distribution $\sigma_i(\Theta)$ is given by the convolution product of the basic charge distribution $\gamma_p(\Theta)$ with the electrode voltages V_i.

As an example the charge density distributions on the periods of the multielectrode periodic Engan's transducers[12] are shown in Fig. 4 for N=2,3,4 and 6 with different polarity sequences and metallization ratio $\eta=0.5$.

2.4. Interrelation of the electrode charges and voltages. Capacitance and potential matrices

By integrating (6) over the electrode width we can deduce the following closed-form relations to express charges in terms of voltages and vice versa:

$$Q_i=\sum_{p=0}^{N-1}\gamma_p V_{i-p},\qquad V_i=\sum_{p=0}^{N-1}\gamma_p^{+} Q_{i-p} \tag{7}$$

where

$$\frac{\gamma_p}{\gamma_p^{+}}=\frac{1}{N}\sum_{s=0}^{N-1}\frac{\tilde{\gamma}_s}{\tilde{\gamma}_s^{+}}\,e^{-j2\pi ps/N} \tag{8}$$

or in the matrix form

$$Q=\Gamma V,\qquad V=\Gamma^{+}Q \tag{9}$$

$$\Gamma=H^{-1}\tilde{\Gamma}H,\qquad \Gamma^{+}=H^{-1}\tilde{\Gamma}^{+}H \tag{10}$$

where "+" denotes pseudo-inversion[13] of the matrix Γ which is degenerate due to the charge neutrality condition. The elements of the diagonal matrices $\tilde{\Gamma}$ and $\tilde{\Gamma}^{+}$ are interrelated as $\tilde{\gamma}_s^{+}=1/\tilde{\gamma}_s$, $s\neq 0$ with $\tilde{\gamma}_0^{+}=0$ where the matrix elements $\tilde{\gamma}_s$ are given by (4). The capacitance matrix Γ relating charges to voltages has the structure

$$\Gamma=\begin{bmatrix}\gamma_0 & \gamma_1 & \gamma_2 & \cdots & \gamma_2 & \gamma_1\\ \gamma_1 & \gamma_0 & \gamma_1 & \cdots & \gamma_3 & \gamma_2\\ \gamma_2 & \gamma_1 & \gamma_0 & \cdots & \gamma_4 & \gamma_3\\ \vdots & & & & & \vdots\\ \gamma_2 & \gamma_3 & \gamma_4 & \cdots & \gamma_0 & \gamma_1\\ \gamma_1 & \gamma_2 & \gamma_3 & \cdots & \gamma_1 & \gamma_0\end{bmatrix} \tag{11}$$

and the pseudo-inverse potential matrix Γ^{+} relating voltages to charges has the same structure.

Quantities $\gamma_p=\gamma_p(N,\eta)$ are charges on the electrodes of the basic periodic structure with the only electrode activated. They are slowly varying functions of the metallization ratio η and fall off rapidly (at the rate of the order $1/p^2$) as the relative electrode number p increases. The coefficients γ_p have the simplest form for a special case of $\eta=0.5$ when $\tilde{\gamma}_s=2\varepsilon\sin\pi s/N$. Then formula (8) is converted to the simple analytic expression after closed-form summation

$$\gamma_p=2\varepsilon\frac{\sin p/N}{N(\cos 2\pi p/N-\cos p/N)}, \qquad p=0,\ 1,\ \ldots,\ N-1. \tag{12}$$

The known results[8] follow from Eqs. (4) and (8) as a particular limiting case of $N\rightarrow\infty$, with a discrete variable s/N replaced by a continuous variable ν and summation replaced by integration:

$$\gamma_p=\int_0^1\tilde{\gamma}(\nu)e^{-j2\pi p\nu}d\nu, \quad \tilde{\gamma}(\nu)=2\varepsilon\sin\pi\nu\frac{P_{-\nu}(\cos\Delta)}{P_{-\nu}(-\cos\Delta)}, \quad p=0,\ 1,\ 2,\ldots \tag{13}$$

2.5. Static capacitance of SAW transducers

We deduce an analytic expression for the static capacitance C using the energy conservation law written in the form

$$\frac{1}{2}C\Delta V^2=\frac{1}{2}V^*Q \tag{14}$$

where ΔV is the voltage applied to the transducer bus-bars, V and Q are vectors of the potentials and charges on the electrodes, and the asterisk denotes Hermitian conjugation. By substitution the first Eq. (9) into (14) we derive the following expression for the static capacitance of an unapodized SAW transducer of the unit aperture $W=1$

$$C=\frac{1}{\Delta V^2}V^*\Gamma V=\frac{1}{N\Delta V^2}\tilde{V}^*\tilde{\Gamma}\tilde{V} \tag{15}$$

or in the scalar form

$$C=\frac{1}{\Delta V^2}\sum_{i=0}^{N-1}\sum_{k=0}^{N-1}\gamma_{ik}V_iV_k=\frac{1}{N}\sum_{s=1}^{N-1}\tilde{C}_s, \qquad \tilde{C}_s=\tilde{\gamma}_s|\frac{\tilde{V}_s}{\Delta V}|^2 \tag{16}$$

where quantities $\gamma_{ik}=\gamma_{|i-k|}$, $i\neq k$ may be interpreted as the charges on the capacitors $C_p=-\gamma_p$, $p=|i-k|$ between i-th and k-th electrodes (Fig. 5) and γ_0 is equal to the total sum of the charges on all the capacitors.

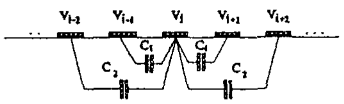

Fig. 5. Interelectrode capacitors

Using the identities

$$2V_iV_k=V_i^2+V_k^2-(V_i-V_k)^2, \quad \sum_{i=0}^{N-1}\gamma_{ik}=\sum_{k=0}^{N-1}\gamma_{ik}=0 \tag{17}$$

and taking into account the full transducer aperture W formula (16) may be converted to the form

$$C=-\frac{1}{2}\sum_{i=0}^{N-1}\sum_{k=0}^{N-1}\gamma_{ik}W_{ik}=-\sum_{i=0}^{N-1}\sum_{k=0}^{i-1}\gamma_{ik}W_{ik}, \quad W_{ik}=(\frac{V_i-V_k}{\Delta V})^2W. \tag{18}$$

If each electrode is connected to either of bus-burs ($V_i=\pm\Delta V/2$) then $W_{ik}=0$ for $V_i=V_k$ and $W_{ik}=W$ for $V_i\neq V_k$. Thus, the partial apertures W_{ik} are equal to the overlaps of the i-th and k-th electrodes.

The expression (18) is converted to the compact form by using the symmetry properties and reordering the summation

$$C=\sum_{p=1}^{N-1}C_pL_p, \quad L_p=\sum_{k=0}^{N-1-p}W_{k,k+p} \tag{19}$$

where quantities L_p are total overlaps of all the nearest, next nearest neighbour electrodes, and so on respectively. Therefore, static capacitance is composed of the weighted interelectrode capacitors, with formula (19) being applicable both to unapodized and apodized SAW transducers with arbitrary polarity sequences.

The known Engan's results for the capacitance of multielectrode periodic transducers[12] follow from Eq. 16 as a special case for N=2,3,4 and 6. Capacitance values per period are shown in Fig. 6 for metallization ratio $\eta=0.5$.

2.6. Mixed electrostatic problem solution

Given a priori all the electrode voltages, the charges on the electrodes can be calculated from the first Eq. (9) using the capacitance matrix Γ and vice versa if all the electrode charges are known the voltages can be recovered from the second Eq. (9) using the pseudo-inverse potential matrix Γ^+.

N	V_i	$\bar{\gamma}_1/\epsilon$	$\lvert\tilde{V}_1/\Delta V\rvert^2$	C/ϵ
2	+ -	2	1	1
3	+ - +	$\sqrt{3}$	1	$2/\sqrt{3}$
	+ 0 -	$\sqrt{3}$	3/4	$\sqrt{3}$
4	+ + - -	$\sqrt{2}$	2	$\sqrt{3}/2$
	+ 0 - 0	$\sqrt{2}$	1	$1/\sqrt{2}$
6	+ 0 - - 0 +	1	3	1

Fig. 6. Capacitance per period for periodic multielectrode Engan's transducers

Now consider more general mixed electrostatic problem where each electrode may be characterized either by its potential or its charge prescribed a priori.

Rewrite the first Eq. (9) in the following block-matrix form

$$\begin{bmatrix} Q_0 \\ Q_1 \end{bmatrix} = \begin{bmatrix} \Gamma_{00} & \Gamma_{01} \\ \Gamma_{10} & \Gamma_{11} \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \end{bmatrix} \tag{20}$$

where V_0, Q_1 are vectors of the prescribed voltages and charges and Q_0, V_1 are vectors of the unknown charges and voltages. It is assumed in (20) that vector and matrix elements are properly permutated and renumbered.

The solution of the matrix equations (20) with respect to the unknown vectors Q_0 and V_1 is given by

$$\begin{bmatrix} Q_0 \\ V_1 \end{bmatrix} = \begin{bmatrix} \Gamma_{00}-\Gamma_{01}\Gamma_{11}^{-1}\Gamma_{10} & \Gamma_{01}\Gamma_{11}^{-1} \\ -\Gamma_{11}^{-1}\Gamma_{10} & \Gamma_{11}^{-1} \end{bmatrix} \begin{bmatrix} V_0 \\ Q_1 \end{bmatrix} \tag{21}$$

where the vectors Q_0 and V_1 are expressed in terms of the a priori prescribed vectors V_0 and Q_1. An iterative Gauss-Seidel procedure[14] may be applied instead of the matrix Γ_{11} inversion, with the storage requirements considerably reduced as only $N/2$ different coefficients γ_p should be stored.

2.7. SAW transducers with floating strips

The solution for SAW transducers with separate or interconnected (sectioned) floating strips may be treated as a special case of the aforementioned mixed electrostatic problem, with the charge neutrality and equipotentiality conditions imposed to the floating sections.

Rewrite the first Eq. 9 in the following partitioned form

$$\begin{bmatrix} Q_0 \\ Q_1 \\ \cdot \\ \cdot \\ Q_M \end{bmatrix} = \begin{bmatrix} \Gamma_{00} & \Gamma_{01} & \cdots & \Gamma_{0M} \\ \Gamma_{10} & \Gamma_{11} & \cdots & \Gamma_{1M} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \Gamma_{M0} & \Gamma_{M1} & \cdots & \Gamma_{MM} \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ \cdot \\ \cdot \\ V_M \end{bmatrix} \tag{22}$$

where Q_0 and V_0 are vectors of the charges and voltages on the connected fingers and subvectors Q_i and V_i, $i=1, M$ with the elements Q_k^i and V_k^i, $k=1, N_i$ contain the charges and voltages on the electrodes of the floating sections, with M being a total number of the floating sections including the separate floating strips and N_i being a number of the electrodes in the i-th section.

By applying the charge neutrality condition

$$L_i^T Q_i = 0 \quad \rightarrow \quad \sum_{k=1}^{N_i} Q_k^i = 0, \quad i=\overline{1,M} \tag{23}$$

and the equipotentiality condition

$$\Phi_i L_i = V_i \quad \rightarrow \quad V_k^i = \Phi_i, \quad i=\overline{1,M}, \quad k=\overline{1,N_i} \tag{24}$$

to each floating section where $\Phi=[\Phi_1\ \Phi_2\ \ldots\ \Phi_M]^T$ is the vector of the unknown voltages on the floating sections and each vector $L_i=[1\ 1\ \ldots\ 1]^T$ contains N_i unity elements, the initial Eq. 22 may be convolved to the following reduced form

$$\begin{bmatrix} Q_0 \\ 0 \end{bmatrix} = \begin{bmatrix} \Im_{00} & \Im_{01} \\ \Im_{10} & \Im_{11} \end{bmatrix} \begin{bmatrix} V_0 \\ \Phi \end{bmatrix} \tag{25}$$

where $\Im_{ik}$ are the appropriate matrix-blocks of the reduced capacitance matrix $\Im$ and the zero vector $0=[0\ 0\ \ldots\ 0]^T$ of length

M contains zero charges on the floating sections. Solving the second Eq. (25) with respect to the unknown vector Φ leads to

$$\Phi = -\mathfrak{Z}_{11}^{-1}\mathfrak{Z}_{10} V_0 \qquad (26)$$

i.e. the unknown voltages Φ_i on the floating sections are expressed in terms of the prescribed voltages V_k on the connected fingers.

As an example the charge density distribution on the electrodes of the hypothetic generalized SAW transducer comprising both separate and interconnected floating strips is shown in Fig. 7, with unknown potentials on the floating strips calculated using Eq. 26. The results of calculations are in excellent agreement with those obtained by the numeric method of moments[4].

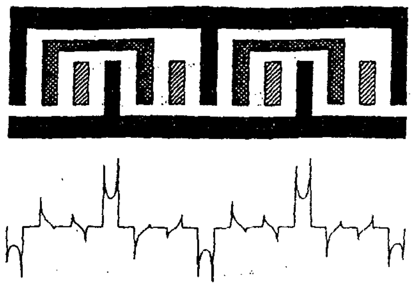

Fig. 7. Hypothetic generalized SAW transducer with floating strips

3. CONCLUSION

The complete closed-form solution of the electrostatic problem for generalized periodic SAW transducers has been considered in this paper. A feature of the present approach is that a real transducer is treated as one generalized period in an infinite periodic array derived by sequential multiple repetition of the initial transducer. In general case a transducer may contain separate or interconnected (sectioned) floating strips. Special uncoupling grounded or disconnected electrodes are introduced to uncouple between adjacent periods while calculating and/or to suppress end effects in a finite length transducer. Unknown potentials on the floating electrodes are found by the appropriate partition and solving the block-matrix equations.

Some known results for infinite periodic arrays follow from the theory as a particular limiting case of the infinite period. However, the advantages of the proposed approach are apparent:

1. Physical redundancy of the infinite period is removed.
2. There is a simple criterion (uncoupling between adjacent periods) to evaluate a number of the uncoupling (guard) electrodes to be introduced in a real transducer.
3. Closed-form expressions are derived in terms of finite summation instead of finite series and integrals.

The theory is quite general and may be applied to the polarity- and capacity-weighted transducers, multiphase unidirectional transducers, etc.

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