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FACTORIZATIONAL SYNTHESIS OF SAW BANDPASS FILTERS

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Abstract: Design of surface acoustic wave (SAW) filters with linear phase (LP), prescribed nonlinear phase (NLP), minimum phase (MP), and optimum minimum phase (OMP) characteristics by factorizing (splitting) the filter frequency response is considered. The universal factorization algorithm based on the Z-transform roots searching, lexicographical sorting, and sharing between the transducers is proposed in the paper. A feature of the algorithm is that irrespective of the overall phase specifications both weighted IDT tend to have minimum phase characteristics being less sensitive to the tap-weighting inaccuracies.

The design examples of LP, NLP, MP, and OMP SAW filters are presented which confirm the flexibility and efficiency of the design procedure.

Introduction

In the first order approximation the frequency response (FR) of a SAW filter can be written as the following product [1]

$$F(\omega)=\xi(\omega)\,F_1(\omega)\,F_2^*(\omega) \qquad (1)$$

where the functions $F_i(\omega)$, $i=1,2$, are attributed to the IDT array factors, and the asterisk denotes the complex-conjugation. The skewing factor $\xi(\omega)$ is introduced to account for the IDT element factors and/or the multistrip coupler FR, etc. Without the loss of generality we may suppose hereafter $\xi(\omega)=1$ as this factor can be implicitly accounted for by a proper predistortion of the function $F(\omega)$.

To simplify the design the frequency response $F_1(\omega)$ of one of the transducers is usually supposed to be given a priori while the other's $F_2(\omega)$ is optimized to satisfy the filter transfer function specifications [2, 3].

An alternative design simplification is to use two identical weighted IDT with the "halved" frequency response

$$F_1(\omega)=F_2^*(\omega)=\sqrt{F(\omega)}\ . \qquad (2)$$

But in general case both functions $F_1(\omega)$ and $F_2(\omega)$ are to be found separately from the design, any a priori restrictions to them resulting in a filter length increase.

For linear phase SAW filters composed of two weighted IDT in conjunction with the multistrip coupler the factorizational synthesis technique [4-6] based on the Z-transform roots searching and sharing was elaborated. The present paper generalizes the factorization technique to the NLP design including minimum phase SAW filters.

Basic Z-transform Properties of a SAW Filter Response

Supposed the functions $F_i(\varphi)$, $i=1,2$ to be the complex trigonometric (exponential) polynomials of the order N_i, the overall transfer function $F(\varphi)$ is also the trigonometric polynomial of the order $N=N_1+N_2$-1, i.e.

$$F(\varphi)=F_1(\varphi)F_2^*(\varphi)=\sum_{k=0}^{N-1}A_k e^{jk\varphi} \qquad (3)$$

where $\varphi=\pi\omega/\omega_\pi$ is an angle variable, ω_π being a transducer synchronous frequency. The complex trigonometric polynomial (3) can be converted to the algebraic polynomial $F(z)$ using Z-transform [7]

$$F(z)=F_1(z)F_2(1/z)=\sum_{k=0}^{N-1}A_k z^k,\ z=re^{j\varphi}\ . \qquad (4)$$

The frequency response $F(\varphi)$ is a Z-transform $F(z)$ evaluated on the unit circle $z=e^{j\varphi}$.

According to the fundamental theorem of algebra the scaled polynomial (4) can be expressed in terms of their roots z_i as follows

$$F(z)=\prod_{i=1}^{N-1}(z-z_i)=\prod_i M_i(z)\prod_j D_j(z)\prod_k Q_k(z) \qquad (5)$$

where for the real-valued coefficients A_k the roots (zeros) $z_i=r_i e^{j\varphi_i}$ appear in the complex-conjugated pairs (couples)

$$D_j(z)=D(z,z_j)=(z-z_j)(z-z_j^*),\quad \varphi_j\neq\pi n \qquad (6)$$

or degenerate into the single zeros (monozeros) on the real axis of the Z-plane

$$M_i(z)=M(z,z_i)=z-z_i,\quad \varphi_i=\pi n. \qquad (7)$$

Moreover, to ensure a phase linearity each zero couple at the point z_i off the unit circle ($r_i\neq 1$, $\varphi_i\neq\pi n$) must have its reciprocal at the point $1/z_i$ with the same scaled magnitude response but with the phase reversal composing a linear phase quadruplet

$$Q_k(z)=Q(z,z_k,1/z_k)=D(z,z_k)D(z,1/z_k)= =(z-z_k)(z-z_k^*)(z-1/z_k)(z-1/z_k^*) \qquad (8)$$

when combined together (Fig. 1).

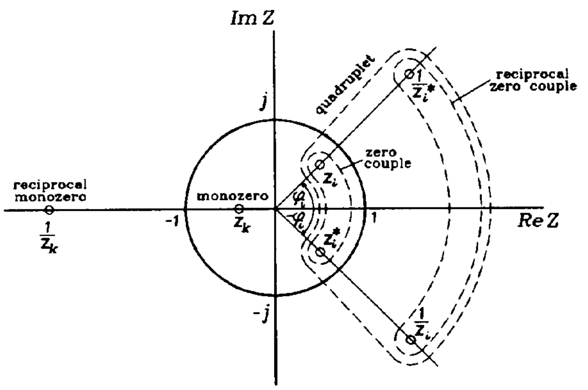


Fig. 1. Z-transform monozeros, couples of zeros, and linear phase quadruplet

Thus, the zeros distribution on the Z-plane depends highly on the SAW filter magnitude shape and phase characteristic required. While in the general case of the NLP filter zero couples $D_i(z)$ may occur both inside as well as outside the unit circle providing the desired magnitude and phase approximation, all zeros off the unit circle must occur in quadruplets to ensure a phase linearity. Minimum (or maximum) phase SAW filters have zeros inside (or outside) the unit circle. Zero couples on the unit circle correspond to the real roots of the overall transfer function $F(\varphi)$ at the points φ_i and always have linear phase response.

The factorized form (5) allows to observe visually how each zero contributes to the overall transfer function. One could effectively control a SAW frequency response by the withdrawal, substitution, and addition of zeros.

An illustrative example of the elemental root frequency response $D_i(\varphi)$ is shown in Fig. 2 for $r_i=0.5$, 0.75, and 1.0 where $\varphi_i=\pi/4$. One can see that moving a zero z_i off the unit circle results in the deterioration of the outofband attenuation.

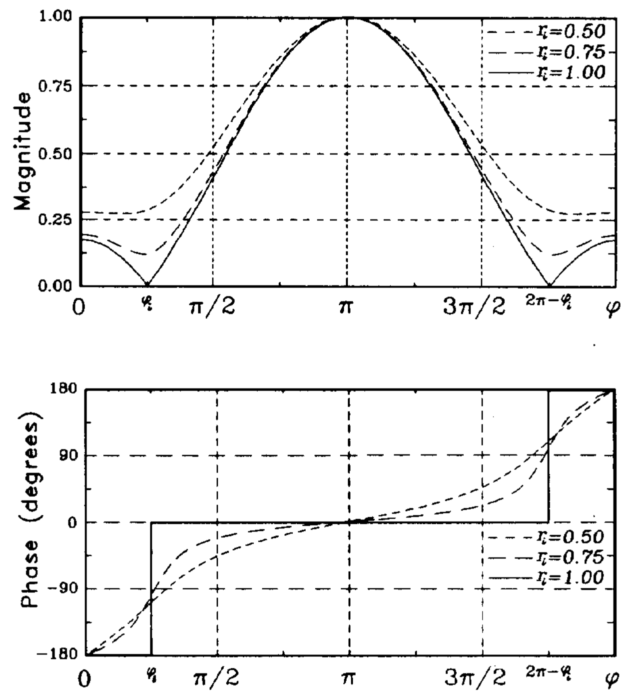


Fig. 2. Frequency response $D_i(\varphi)$ of the elemental zero couple

Factorizational Synthesis Algorithm

The ultimate purpose of the algorithm is to determine the separate transducer responses $F_1(\varphi)$, $F_2(\varphi)$, and the transducers tap-weights from the overall transfer function $F(\varphi)$.

The design procedure starts from the optimization of the function $F(\varphi)$ to meet the desired magnitude and phase specifications. The known FIR synthesis techniques, for example the Remez exchange algorithm [7] or linear programming [8] may be directly applied to the LP design without any adaptations. But to meet the supplementary phase or group delay requirements one should optimize separately the real and image part using one of the tolerance transformation scheme [8-11] or resort to nonlinear programming [12].

The functions $F_1(\varphi)$ and $F_2(\varphi)$ so far unknown can be found from (5) by sharing the roots z_i in the systematic manner between IDT. There are many possible ways of factorizing but one should take into consideration the following:

1) a stopband attenuation depends on the zeros distribution density. The closer are the roots allocated on the unit circle, the higher is a stopband attenuation;

2) moving the root off the unit circle improves the passband shape factor but deteriorates the stopband attenuation.

Hence, while factorizing it is preferable to maintain in the stopband nearly the even zeros density on (or near) the unit circle and to minimize the magnitude difference of the transducers elemental root factors in the passband.

The following factorization algorithm may be applied.

1. Find Z-transform roots $z_i=e^{j\varphi_i}$ using the root solving program for high-order polynomials [13].

2. Sort the roots lexicographically in their angles φ_i and radii r_i.

3. Assign every second root in the filter stopband and all passband roots inside the unit circle to the input IDT, the others being attributed to the output IDT.

4. Restore the coefficients of the complex trigonometric polynomials $F_1(\varphi)$ and $F_2^*(\varphi)$ by using the multiple recurrent convolution of the elemental root factors $M_i(\varphi)$ and $D_j(\varphi)$ or by performing the discrete Fourier transform.

5. Invert in time the coefficients of the output IDT to account for the complex-conjugation of the function $F_2(\varphi)$ in the expression (1).

The algorithm is general and may be applied both LP and NLP designs.

A feature of the factorization technique above is that after the step 5 both IDT have their roots inside or on the unit circle satisfying the minimum phase condition [7]. Such transducers are less sensitive to the tap-weighting inaccuracies [14] if compared to the conventional design where two identical weighted IDT are used.

Another feature is that quadruplets are splitted into inside and outside zero couples with the same scaled magnitude response to be attributed to the input and output IDT. Thus, the minimal difference in the passband magnitude shape for both transducers is attained.

Linear Phase Design

The factorizational synthesis is illustrated by an example of a SAW bandpass LP filter design with the -3 dB fractional bandwidth of 50% and the -3/-40 dB shape factor of 1.5, the filter center frequency being 25 MHz. The design results are presented in Fig. 3 and 4 where for comparison the conventional design results using two identical IDT are also shown. The design specifications are the same for both designs, the stopband attenuation being -60 dB and the passband peak-to-peak ripple being 0.1 dB.

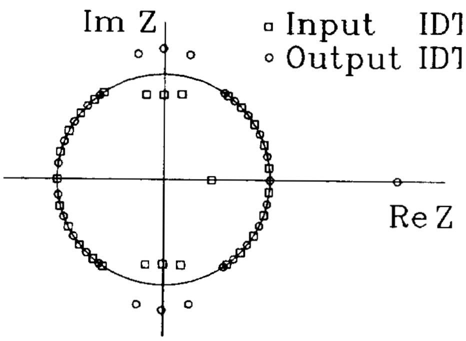
The Z-transform roots distribution is shown in Fig. 3 where bars and circles denote zeros attributed to the input and output IDT respectively. We can see that there are 4 monozeros and 3 LP quadruplets off the unit circle, other 20 LP zero couples allocated on the unit circle in the case of factorizational synthesis. In the case of the conventional design all zeros (4 monozeros, 3 quadruplets, and 12 couples of zeros) are double (Fig.3b).

The filter frequency responses versus a normalized frequency $\nu=\varphi/2\pi=\omega/2\omega_\pi$ are shown in Fig. 4 where the curves 1 and 2 correspond to the responses $F_1(\omega)$ and $F_2(\omega)$ of the input and output IDT, the curve 3 being the filter transfer function $F(\omega)=F_1(\omega)F_2^*(\omega)$.

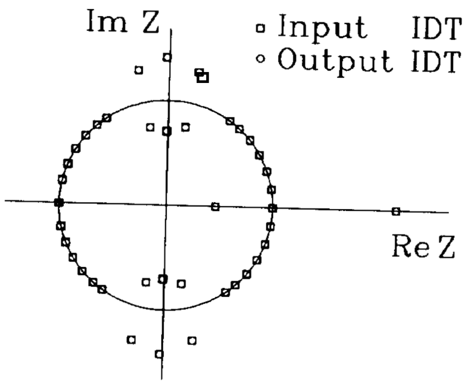
At the same approximation accuracy the filter length is $N=57$ ($N_1=N_2=29$) in the factorizational synthesis and $N=81$ ($N_1=N_2=41$) in the conventional design. In both cases the IDT synchronous frequency $\omega_\pi=2\omega_0$ that corresponds to the IDT structures with splitted electrodes, ω_0 being the filter central frequency.

The gain in the filter length due to the factorization is of 30% for this example if compared to the conventional design. The shorter filter length is due to a more rational arrangement of zeros and extremuma in a filter stopband (Fig. 4). Namely, extremuma of one transducer are allocated in the proximity of zeros of the other transducer and vice versa while all extremuma and zeros are superimposed each other in the conventional design.

In the case of factorizational synthesis both transducers have identical minimum phase group delay time characteristics (Fig.4a) to cancel each other in the overall filter frequency response due to the complex-conjugation. Contrary to that, the transducers have linear phase characteristics in the conventional design.

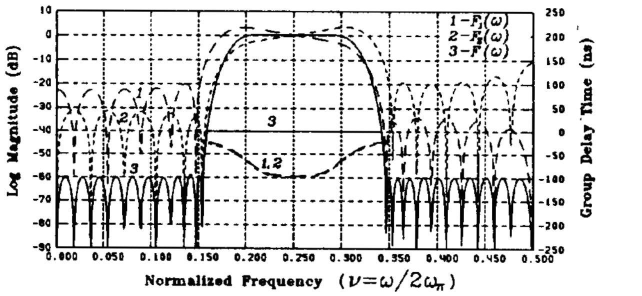


(a) factorizational synthesis

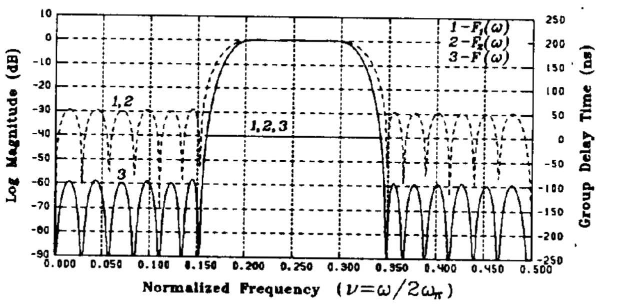


(b) conventional design

Fig. 3. Z-transform roots distribution of a SAW bandpass LP filter



(a) factorizational synthesis



(b) conventional design

Fig. 4. Frequency response of a SAW bandpass LP filter

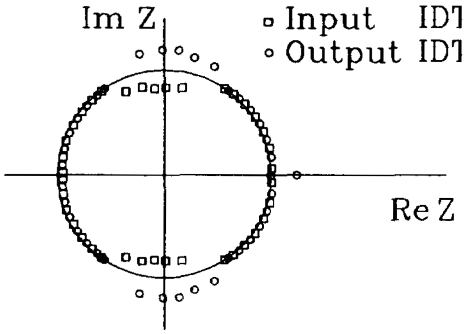
Nonlinear Phase Design

The nonlinear phase design is illustrated by the factorizational synthesis of a dispersive (chirp) SAW bandpass filter with the same magnitude specifications as in the previous example.

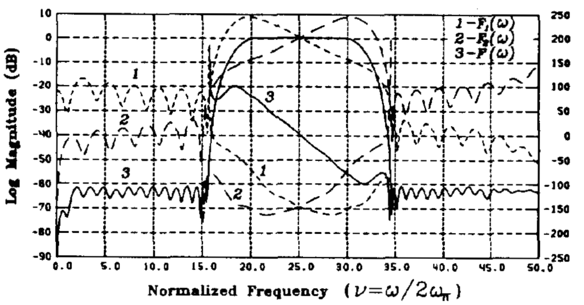
The design results are presented in Fig. 5 where the advanced tolerance transformation scheme [11] was used for the NLP filter transfer function synthesis.

Due to the supplementary group delay time requirements the filter length was increased to $N=77$, the transducer lengths being $N_1=N_2=39$. Contrary to the previous examples, the transducers have different group delay time characteristics to ensure required approximation, the group delay time passband approximation error being less than 5 ns (Fig. 5b).

The uncompensated zero couples off the unit circle causes the transducer responses $F_1(\varphi)$ and $F_2(\varphi)$ to be highly asymmetrical in the filter passband if compared to the previous examples.



(a) Z-transform roots distribution

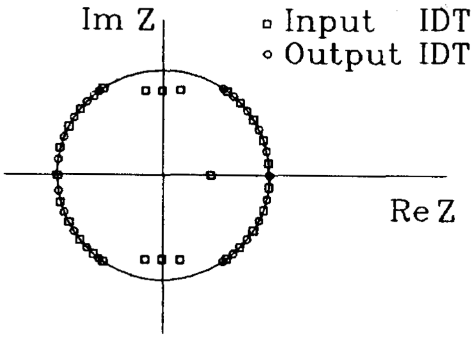


(b) frequency response

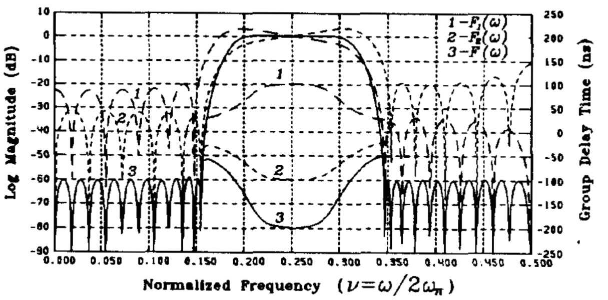
Fig. 5. Design results of a chirp SAW bandpass filter

Minimum Phase SAW Filter Design

Due to the intrinsic minimum phase feature peculiar to the factorization algorithm proposed it may easily be adopted to the minimum phase SAW filter design starting from the LP or NLP prototypes. Before factorizing one should only transform all the outside roots into their reciprocals inside the unit circle. As there will be no outside zeros now, a modified factorization algorithm should be applied where at the step 3 every second root irrespective of its location is assigned to the input IDT, the others being attributed to the output IDT. Virtually, starting from the LP prototype the same result could be obtained by omitting the last step of the factorization algorithm above.



(a) Z-transform roots distribution



(b) frequency response

Fig. 6. Design results of a SAW bandpass MP filter

A MP response and its prototype always have equal filter length and exactly the same magnitude response that supplies an excellent vehicle to compare their properties if necessary. A MP design allows to decrease the filter group delay time that could prove to be useful in some applications.

The Z-transform roots distribution and the frequency response of the MP filter designed from the LP prototype (Fig. 4a) are shown in Fig. 6.

The filter has 3 double passband zero couples and one double monozero inside the unit circle, the others being allocated on it. As in the case of LP design the filter length is $N=57$, the transducer lengths being $N_1=N_2=29$. The transducers have the opposite group delay characteristics to be summed up in the overall transfer function due to the complex-conjugation that results in minimizing the filter group delay at the central frequency.

Optimum Minimum Phase SAW Filter Design

A minimum phase design above is not optimum. There is a way to reduce further the MP filter length by applying the more sophisticated procedures [15-17] of the optimum MP design.

The major design steps are: 1) the synthesis of the squared optimum magnitude LP (OMLP) prototype $|F(\varphi)|^2$ of the order $2N$-1, all its double zeros being off and on (or near) the unit circle; 2) the synthesis of the optimum minimum phase response $F(\varphi)$ by retaining zeros inside or on the unit circle if available; 3) splitting the function $F(\varphi)$ into the separate IDT responses $F_1(\varphi)$ and $F_2(\varphi)$ using the modified factorization algorithm above.

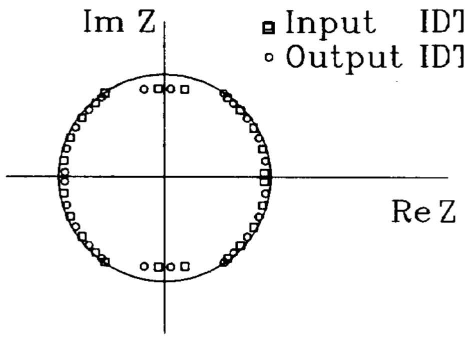
The squared OMLP prototype $|F(\varphi)|^2$ may be synthesized by the Hermann-Shuessler technique [15-17] using the Remez exchange algorithm [7] for example. The modifications of the McClellan's computer program [7] to overcome divergence problems for very high-order trigonometric polynomials are discussed elsewhere [18].

The design results of the OMP SAW filter with the same magnitude specifications as for the LP filter above are presented in Fig. 7. The modified Hermann-Schuessler procedure [16] and the McClellan's computer program [7] were used to design squared LP optimum magnitude response prototype.

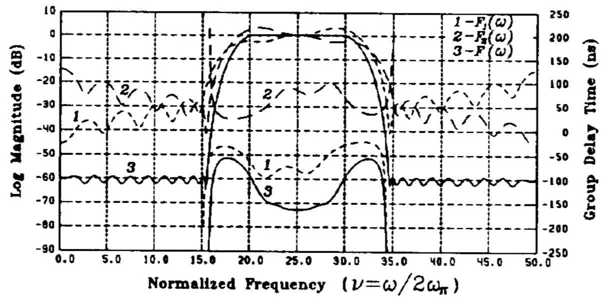
The Z-transform roots distribution after one of each double zeros has been retained is shown in Fig. 7a and the frequency response is presented in Fig. 7b.

At the same design specifications the OMP filter length N=49 is shorter if compared to the previous examples, the transducer lengths being $N_1=N_2=25$. The filter has 4 passband couples of zeros inside the unit circle, other 20 couples of stopband zeros being near the unit circle also inside it.

It is worthy to note that apart from the delay at the central frequency the passband group delay time characteristics for optimum and non-optimum design are very close.



(a) Z-transform roots distribution



(b) frequency response

Fig. 7. Design results of a SAW bandpass optimum MP filter

Conclusion

A general design procedure for LP, NLP, MP, and OMP SAW bandpass filters based on the Z-transform roots searching, lexicographical sorting, and sharing between IDT has been proposed in the paper. The factorization algorithm results in the minimum phase transducers that are less sensitive to the tap-weighting inaccuracies. The problems of the factorizational synthesis of the SAW filters with various phase requirements has been discussed.

Given the design specifications, the factorizational design leads uniquely to the minimum-length SAW filters that are usually of 20-30% shorter if compared to the conventional design where two identical weighted IDT are used.

The filter length may be further reduced in the optimum MP filters. It may be recommended to use such filters in the applications where the dispersionless requirements are not too severe. Moreover, a reasonable compromise between LP and MP designs may be attained in the quasi-minimum phase filters containing a small portion of zeros outside the unit circle.

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